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Rolling Tachyon and S-brane Actions

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hep-th/0211090

w/ Pei-Ming Ho & John Wang (NTU)

hep-th/030????

w/ S. Nagaoka (Komaba), Pei-Ming Ho & John Wang (NTU)

Open string tachyon condensation is established via Sen's conjectures
to be a very important tool for describing D-branes.

It opens up possibilities to describe
dynamical creation and annihilation processes of D-branes.
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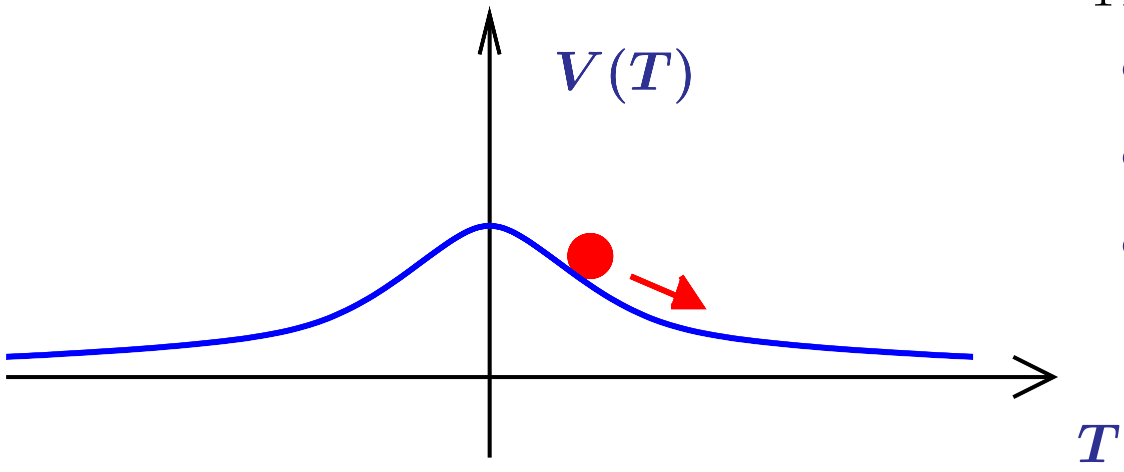
Rolling tachyons

It is known that spacelike-branes (**S-branes**) appear in the rolling tachyon.

——— After reviewing some, I would like to present ———

- **Derivation of S-brane actions**
- **Application of S-brane actions :**
  - **Formation of a closed string in rolling tachyon**
  - **Formation of a lower-diemnsional D-brane**
  - **D-brane as a boosted S-brane**

Rolling Tachyon



The initial papers by A. Sen:

- "Rolling Tachyon" (hep-th/0203211)
- "Tachyon Matter" (hep-th/0203265)
- "Field Theory of Tachyon Matter" (hep-th/0204143)

Sen studied how a non-BPS D-brane decays dynamically, with time-dependent tachyon condensation. His approach to the dynamical decay of unstable D-branes:

- Treating *classical* decay without coupling to closed strings.
- Using *SFTs*, *deformed CFTs* and *boundary states*.

Results:

- No oscillation around the potential minimum, but everlasting rolling toward it.
- End product is pressure-less non-interacting dust with finite energy density, which is called "tachyon matter."
- Proposal of an effective field theory for the tachyon matter.

Tachyon Action

Start with an effective Lagrangian of a non-BPS D(p+1)brane:

$$S = -\int d^{p+2}x \, V(T)\sqrt{1+(\partial_\mu T)^2},$$

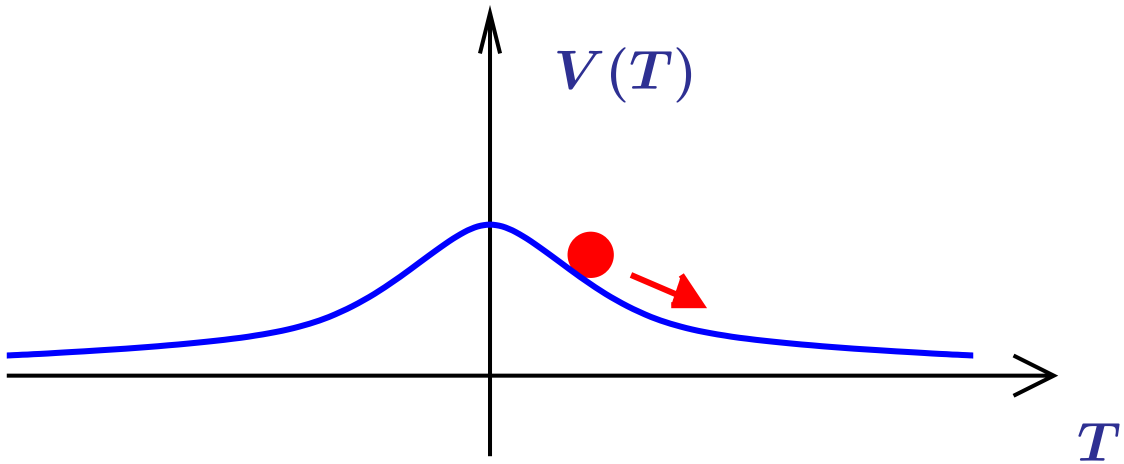
where the potential is the run-away type for large T,

$$V(T) = \exp[-\alpha T], \quad \alpha = 1, \sqrt{2}.$$

- Proposed by [Garousi](0003122) and [Bergshoeff-deRoo-deWit-Eyras-Panda](0003221), so that it is consistent with scattering amplitudes and T-duality.
- [Sen](0203265,0204243) used this Lagrangian to show the appearance of the rolling tachyon classically,

$$\dot{T} = 1 \ (x^0 \to \infty)$$

and reproduction of the exponential fall off of the pressure.



- BSFT reproduces Sen's results mostly. [Sugimoto-Terashima](0205085), [Minahan](0205098)

Sp-brane Solution

[Gutperle-Strominger](0202210)

S$\boldsymbol{p}$-brane = Time-dep. kink in non-BPS D$\boldsymbol{(p+1)}$-brane

Assume a homogeneous tachyon: $T = T(x^0)$

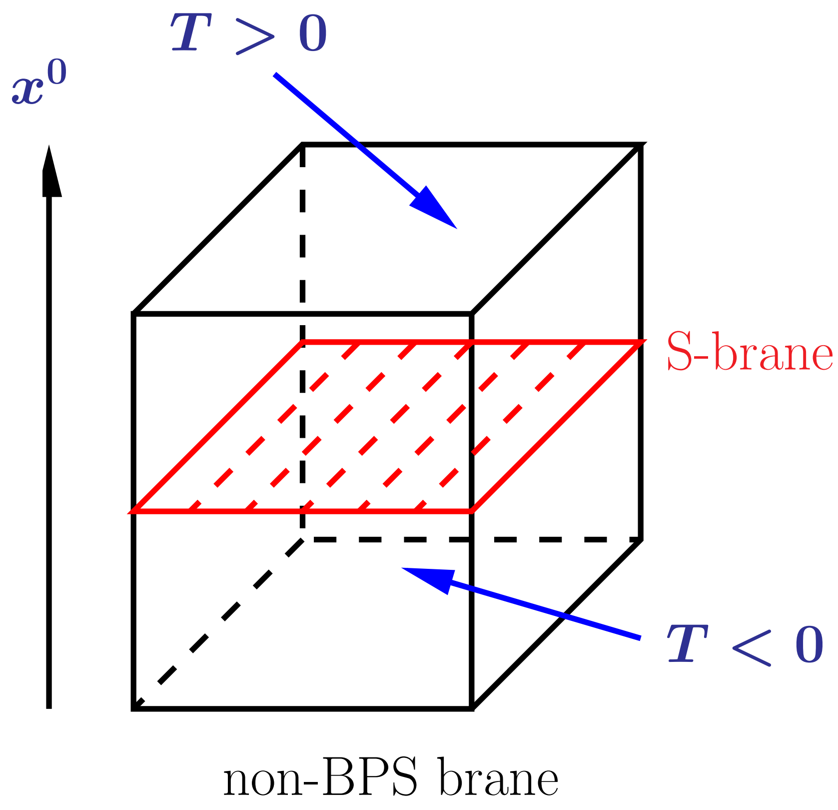
↓

Conserved energy: $\mathcal{E} = \frac{V(T)}{\sqrt{1-\dot{T}^2}} (> V(0))$

↓

A homogeneous solution $T_{\rm cl}(x^0)$:

$$x^0 = \int_0^{T_{\rm cl}} \frac{dT}{\sqrt{1 - V(T)^2/\mathcal{E}^2}}$$



- $V(T) \to 0 \ (T \to \infty)$
  $\Rightarrow \quad T \sim x^0$ (rolling tachyon)
- An Sp-brane sits at $x^0 = 0$ where $T_{\rm cl}(x^0)$ passes its potential maximum $T = 0$.
- This S-brane has a RR-charge. $\quad S_{\rm CS} \propto \int e^{-T^2} dT \wedge C$

  $\longrightarrow$ S-brane $\sim$ Spacelike D-brane

What is the S-brane "dynamics"?
What is the role of the S-branes in string theory?
$\downarrow$
A hint : Low energy effective actions for S-branes — S-brane actions

S-brane actions (1): Lowest order in a scalar field

Zero mode associated with the breaking of the time translation :

$$T = T_{\rm cl}(x^0) + t(x^\mu) \text{ with } t(x^\mu) = X^0(x^{\hat\mu})\dot{T}_{\rm cl}(x^0)$$
$$(\hat\mu = 1, 2, \cdots, p+1)$$

$\downarrow$ Substitution to the tachyon action

$$S = -\mathcal{T}(\mathcal{E}) \int d^{p+1}x^{\hat\mu}\, \frac{1}{2}(\partial_{\hat\nu} X^0)^2$$

S-brane effective action to the lowest order.

- Action is defined on a $p+1$ dimensional Euclidean space.
- X^0 : Scalar field for displacement along time direction.

S-brane actions (2) : Higher order and DBI

Step 1. The dependence on zero mode $\boldsymbol{X^0}$ in tachyon action is

$$S = -\int dx^0 d^{p+1}x \; L\left(T_{\rm cl}\left(\frac{x^0 + X^0(x^{\hat{\mu}})}{\beta(X^0)}\right)\right).$$

Here $\beta(X^0)$ can be fixed by the global Lorentz invariance in the world volume spacetime.

[Arutyunov-Frolov-Theisen-Tseytlin](0012080)

Step 2. We turn on constant gauge field strength as a background on the non-BPS brane.

It appears in the tachyon as

- $\eta^{\mu\nu}$ is replaced by open string co-metric $G^{\mu\nu} = \left((\eta + F)^{-1}_{\rm sym}\right)^{\mu\nu}$.
- BI factor $\sqrt{-\det(\eta + F)}$ is multiplied on the Lagrangian.
- For the tachyon EOMs to be satisfied with the homogeneous profile, $G^{00} = -1,\; G^{0\hat{\mu}} = 0$. (Electric fields $= 0$)

Step 3. Lorentz boosts preserving the open string metric :

$$\Lambda_{\mu}{}^{\nu} G_{\nu\rho} (\Lambda^t)^{\rho}{}_{\sigma} = G_{\mu\sigma}, \quad \Lambda_{\mu}{}^{\nu} = \begin{pmatrix} 1/\beta & \partial_{\hat{\mu}} X^0/\beta \\ * & * \end{pmatrix}$$

$$\Rightarrow \quad \beta = \sqrt{1 - G^{\hat{\mu}\hat{\nu}} \partial_{\hat{\mu}} X^0 \partial_{\hat{\nu}} X^0}$$

Step 4. Performing integration over x^0 gives the S-brane Action :

$$\begin{aligned} S &= S_0(\mathcal{E}) \int d^{p+1}x \; \beta(X^0) \sqrt{\det(\delta + F)_{\hat{\mu}\hat{\nu}}} \\ &= S_0(\mathcal{E}) \int d^{p+1}x \sqrt{\det(\delta_{\hat{\mu}\hat{\nu}} - \partial_{\hat{\mu}} X^0 \partial_{\hat{\nu}} X^0 + F_{\hat{\mu}\hat{\nu}})}. \end{aligned}$$

- Action is defined on a Euclidean worldvolume.
- Kinetic term of X^0 has a "wrong" sign due to that it represents time translation.
- Inclusion of the other transverse scalars is easy:

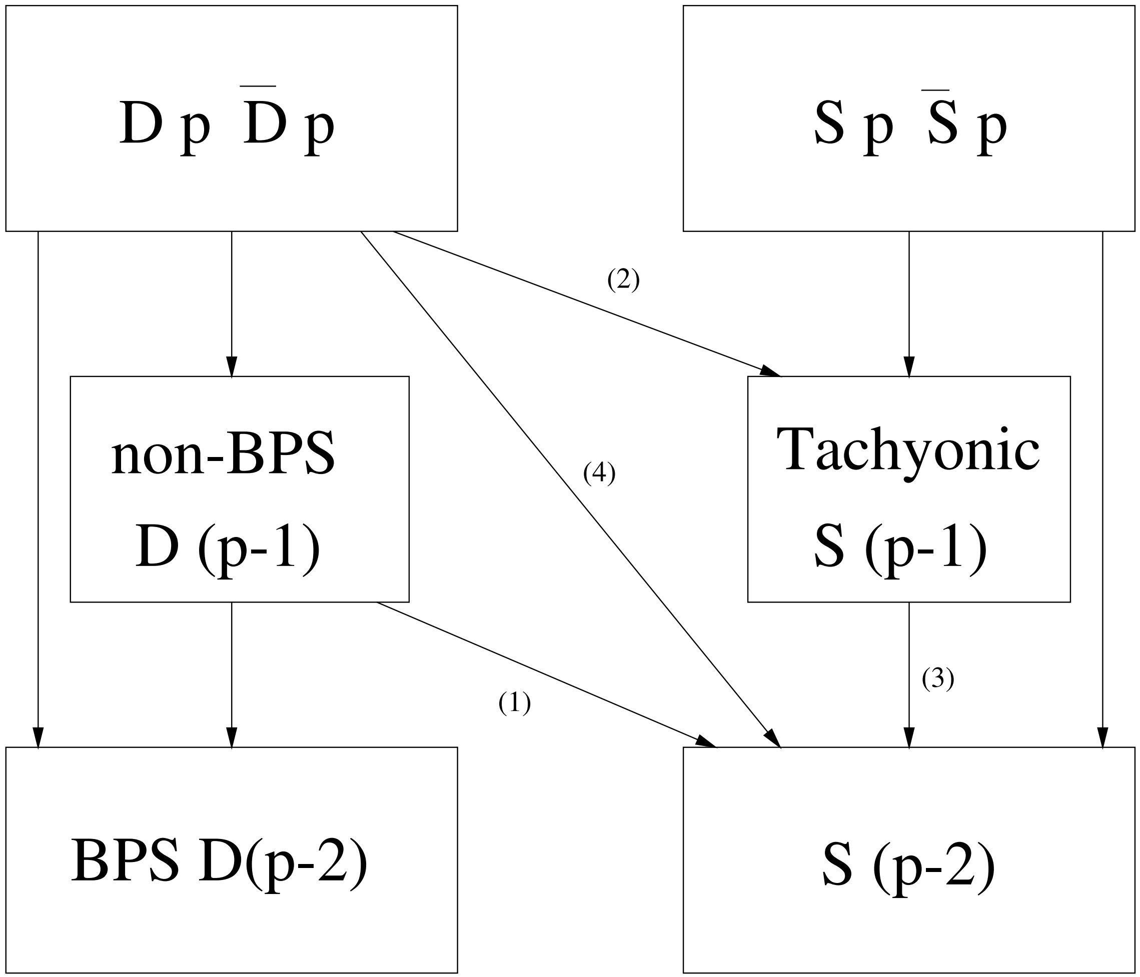
$$S = S_0(\mathcal{E}) \int d^{p+1}x \sqrt{\det(\delta_{\hat{\mu}\hat{\nu}} - \partial_{\hat{\mu}} X^0 \partial_{\hat{\nu}} X^0 + \partial_{\hat{\mu}} X^i \partial_{\hat{\nu}} X^i + F_{\hat{\mu}\hat{\nu}})}.$$

- This suggests that the general form is written with the induced metric

$$S = S_0(\mathcal{E}) \int d^{p+1}x \sqrt{\det(g_{\hat{\mu}\hat{\nu}} + F_{\hat{\mu}\hat{\nu}})}, \quad \text{where} \;\; g_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} X^M \partial_{\hat{\nu}} X^N G_{MN}$$

$\longrightarrow$ Consistnt with CFT derivation.

S-brane Descent Relations



Array (1) : Formation of a time-dependent kink, which is an S-brane.

Array (2) : Formation of a time-dependent kink in $\mathbf{D\bar{D}}$. This indicates a new object which is named as "tachyonic S-brane".

Array (3) and the other vertical short arrows : Formation of an ordinary tachyon kink (space-dependent).

Array (4) : Formation of a space-time vortex in $\mathbf{D\bar{D}}$, which should lead to the S-brane.

Vertical long arrows : Formation of a vortex whose co-dimension is two.

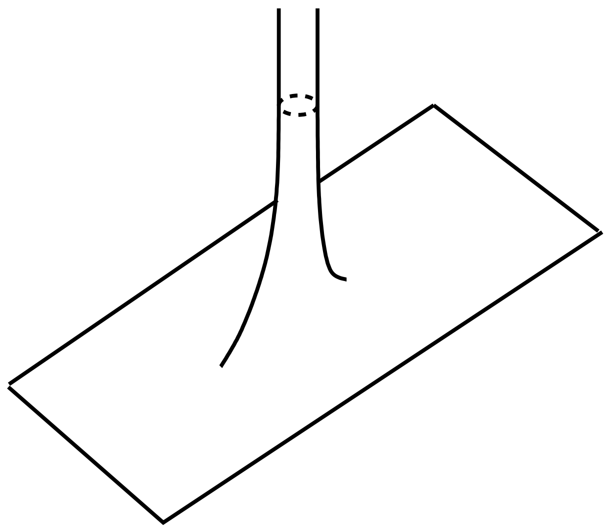
Left and Right sequences are related by Euclideanization.

We will find that S-branes are very useful to describe remnants of tachyon condensation, formation of defects in rolling tachyons.

ex.) [S-branes : $T \sim 0$] $\leftrightarrow$ D-branes and strings in noncommutative tachyons

ex.) Solution of the defect action $\longrightarrow$ Tachyon configuration

[Hirano-K.H.](0102173,0102174)



Solution of a D3-brane action representing a Non-commutative Dirac monopole:

$$X = c/r + \theta_{23} x^1$$
$$B_i = \partial_i X$$
with $r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$

$\downarrow$

Corresponding tachyon configuration:

$$T = uX \text{ with } u \to \infty$$
$$B_i = \partial_i X$$

Also in the S-brane case,
this kind of correspondence is expected.

3-1. Formation of a F-string in rolling tachyon

(1) How the spacelike S-branes can be transformed into timelike?

S-brane action with $\boldsymbol{F}=\mathbf{0}$: $\sqrt{1-(\nabla X^0)^2}$

$$\downarrow$$

$$|\nabla X^0| \le 1 \Leftrightarrow |v| = |\nabla X^0|^{-1} \ge 1$$: S-brane is spacelike.

However, we can make this S-brane timelike as follows.

Turn on a single gauge potential A_{p+1} and suppose that all the worldvolume fields are independent of x^{p+1}.

S-brane action becomes $\sqrt{\det(\delta_{\hat{\mu}\hat{\nu}} - \partial_{\hat{\mu}}X^0\partial_{\hat{\nu}}X^0 + \partial_{\hat{\mu}}A_{p+1}\partial_{\hat{\nu}}A_{p+1})}$

$$||$$

Static DBI with replacement $(X^0, A_{p+1}) \leftrightarrow (A_0, \Phi)$.

$$\Downarrow$$

Spike solutions exist!

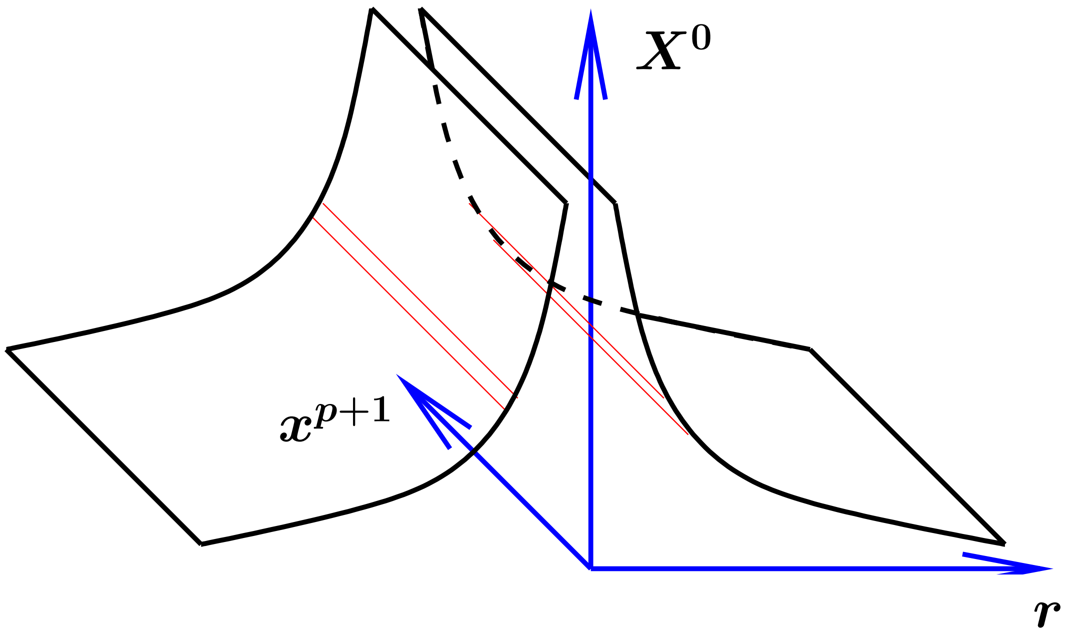
[Callan-Maldacena], [Gibbons]

(2) Spike solution

In the Sp-brane (for $p \geq 3$), the solution is

$$X^0 = A_{p+1} = \frac{c_p}{r^{p-2}} \quad \left(r = \sqrt{(x^1)^2 + \cdots + (x^p)^2}\right)$$

- r is parameterizing also the time evolution of this S-brane.
- As time evolves the radius decreases.
  $\longrightarrow$ Remnant becomes 1+1 dimensional object (string) parametrized by X^0 and x^{p+1}.



- Induced metric on the S-brane:

$$ds^2 = (dx^{p+1})^2 + r^2 d\Omega_{p-1}^2 + \left(1 - \left(\frac{dX^0(r)}{dr}\right)^2\right) dr^2$$

$$\downarrow$$

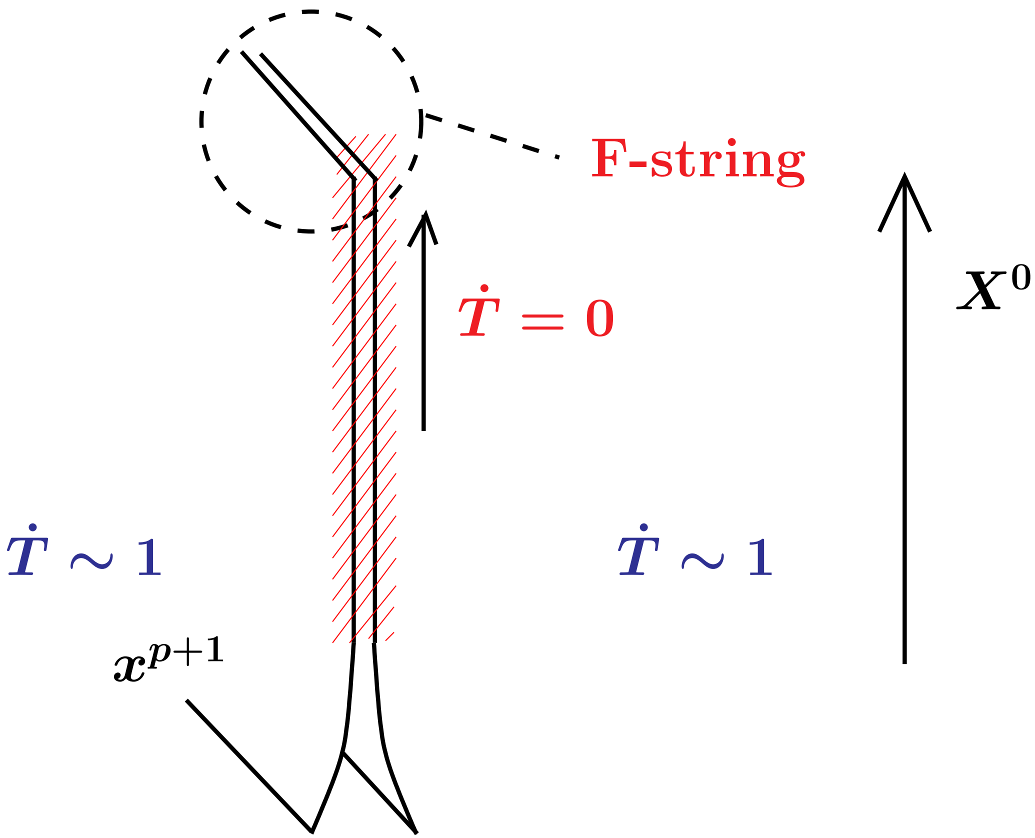
Euclidean for $0 < X^0 < X_c^0$,

Worldvolume is timelike for $X_c^0 < X^0$!

where $X_c^0 \equiv c_p^{1/(p-1)}(p-2)^{(2-p)/(p-1)}$.

(3) The remnant = F-string

[a.] ⟨Induced critical electric field⟩



The field strength induced on the deformed S-brane :

$$F_{0p+1} = \frac{\partial r}{\partial X^0} F_{rp+1} = \frac{1}{2\pi\alpha'}$$

||

Critical electric field
indicating induced F-string charge

This criticality is consistent with rolling tachyon with flux, $\dot{T}^2 + E^2 = 1$ with $\dot{T} = 0$ at late time.

[Sen-Mukdpadhyay] [Gibbons-K.H.-Yi]

⇓

Confined electric flux tube

[**b.**] ⟨F-string tension⟩

In coordinates more appropriate to spacetime viewpoint,

$$\boldsymbol{S = S_0 \int dx^0 d^p x \sqrt{-1 + E_{p+1}^2 + \dot{r}^2}},$$

Canonically conjugate momenta and Hamiltonian :

$$\boldsymbol{D = S_0 \frac{E}{-1+E^2+\dot{r}^2}}, \quad \boldsymbol{P_r = S_0 \frac{\dot{r}}{-1+E^2+\dot{r}^2}}, \quad \boldsymbol{H = \frac{S_0}{\sqrt{-1+E^2+\dot{r}^2}} = \frac{D}{E}}\,.$$

$$\left[\begin{array}{c} \text{Flux quantization} \\ \int \boldsymbol{d^{p-1}x\ D = n} \end{array} \right] + \left[\begin{array}{c} \text{Critical } \boldsymbol{E = 1} \\ \text{at late times} \end{array} \right]$$

$$\downarrow$$

$$\boldsymbol{\int d^p x\ H = \frac{n}{2\pi\alpha'} \int dx^{p+1}}$$

F-string tension

[c.] ⟨Disappearance of D-brane charge⟩

It is natural to take the Ramond-Ramond (RR) coupling of the S-brane to be the same as that for a D-brane. Transforming $\boldsymbol{r}$ into the embedding time $\boldsymbol{X^0}$ we obtain

$$\mu \int C = \mu \int C^{p+1\, r\, \Omega}\, r^{p-1} dx^{p+1} dr\, d\Omega_{p-1}$$
$$= \mu \int C^{p+1\, 0\, \Omega} \left(\frac{X^0}{c_p}\right)^{-\frac{p-1}{p-2}} dX^0 d\chi d\Omega_{p-1}$$

Due to the factor $(\boldsymbol{X^0})^{-(p-1)/(p-2)}$ which goes to zero at late times, we see that this has the D-brane charge which shrinks to zero at the future infinity.

This is similar to the F-string popped-up into a cylindrical D-brane with electric fields, studied by Emparan.

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[a][b][c] $\Rightarrow$ The remnant = F-string

3-2. Formation of a lower dimensional D-brane

(1)

[S-duality of S3-brane] + [Spike solution above]

$\downarrow$

Spike solution representing a formation of
a D-string from a non-BPS D4-brane (an S3-brane)

(S-duality for an S3-brane was already shown by Tseytlin.)

(2) Formation of a D0-brane from an S2-brabe (non-BPS D3-brane)

Lagrangian for a S2-brane with $B_i \equiv \epsilon_{ijk}F_{jk}/2$:

$$L = \sqrt{1 - |\nabla X^0|^2 + |\mathrm{B}|^2 - (\mathrm{B}\cdot\nabla X^0)^2} - i\nabla\chi\cdot\mathrm{B}.$$

$$\rightarrow \quad L = \sqrt{\det(\delta_{ij} - \partial_i X^0 \partial_j X^0 + \partial_i\chi\partial_j\chi)}.$$

$\sim$ SM2-brane

Self-dual configuration of duality-R symmetry $SO(1,7)$:

$$X^0 = \chi = \frac{c}{r}, \qquad r \equiv \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$$

Formation of a D0-brane (= a shrinking dielectric brane)

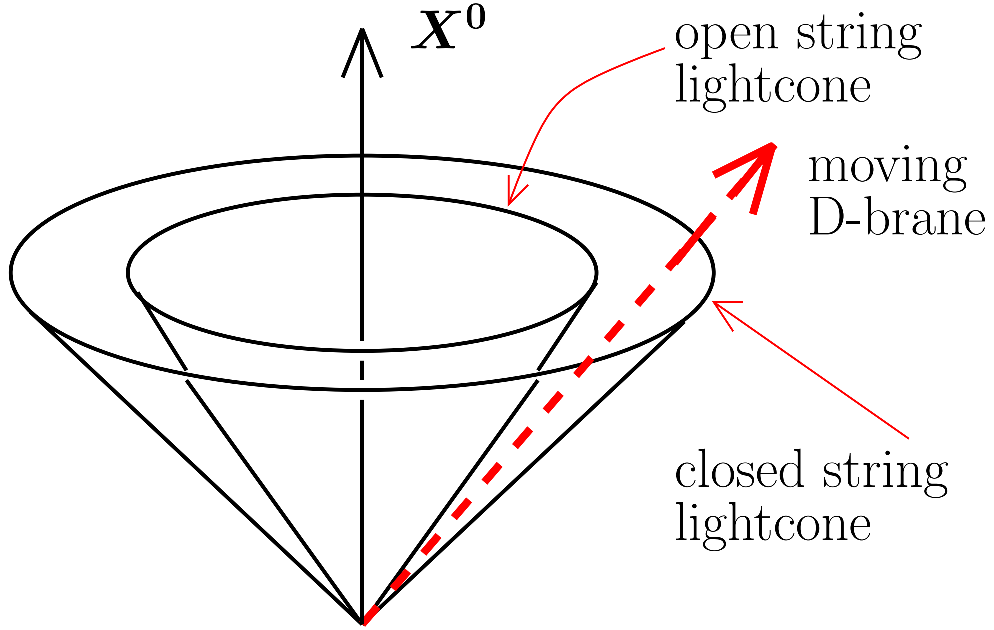
3-3. D-brane as a boosted S-branes

For $\boldsymbol{p = 1}$ we obtain a moving (p,q)-string.

$$\partial_1 X^0 = \frac{c_1}{\sqrt{1 - c_2^2 + c_1^2}}, \quad F_{12} = \frac{c_2}{\sqrt{1 - c_2^2 + c_1^2}}.$$

- The speed along x^1:
$|v_1| = |(\partial_1 X^0)^{-1}| > 1/\sqrt{1 + F_{12}^2}$,
which has a timelike region.

- Why this is possible?
The open string light cone lies always inside the closed string light cone.
[Gibbons-Herdeiro](0008052)



- A boundary state describing this
$\longrightarrow$ it becomes a source for the NS-NS B-field, suggesting that the moving object is a $(\boldsymbol{p}, \boldsymbol{q})$-string.

- Corresponding tachyon solution with
$F_{12} \to \infty$: $T = u x^1$ with $u = \infty$.

- Corresponding tachyon solution

A non-BPS D2-brane : $L = -V(T)\sqrt{-\det(\eta + F)}\,\mathcal{F}\left(G^{\mu\nu}\partial_\mu T\partial_\nu T\right)$

The open string metric with F_{12} is $G_{\mu\nu} = \mathrm{diag}(-1, 1+(F_{12})^2, 1+(F_{12})^2)$

The simplest homogeneous solution : $T = T_{\rm cl}(x^0)$.

Let us make a boost which preserves the metric. A rescaled coordinate $\tilde{x}^1 \equiv \sqrt{G_{11}}x^1$,

$$\begin{pmatrix} x^0 \\ \tilde{x}^1 \end{pmatrix} \rightarrow \begin{pmatrix} x^{0\prime} \\ \tilde{x}^{1\prime} \end{pmatrix} = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \begin{pmatrix} x^0 \\ \tilde{x}^1 \end{pmatrix}.$$

Location of the S1-brane : $x^0 = 0 \quad \longrightarrow \quad x^0 + \tanh\theta\sqrt{G_{11}}x^1 = 0$

The defect is now moving along x^1 with the velocity : $\dfrac{\partial x^1}{\partial x^0} = \dfrac{-1}{\sqrt{G_{11}}\tanh\theta}$.

Take the limit $F_{12} \rightarrow \infty$.

$$T = T_{\rm cl}\left(x^0 + \tanh\theta\sqrt{G_{11}}x^1\right) \quad \longrightarrow \quad T \sim \tanh\theta F_{12} x^1$$

D-brane solution in BSFT

4 Conclusion and Future Directions

Summary

- S-brane actions are constructed.
- Dynamical formation of tachyon remnants is described.
  - Spike solutions exist.
  - They describe formation of a F-string/D-string in a non-BPS D4-brane.
  - D0-brane formation = Shrinking dielectric brane.
  - Moving D-brane = Boosted S-brane.

Future directions

- Much to be studied with use of the S-brane actions!
  - Various brane configurations such as S-brane junctions and spherical S-branes
  - Their supersymmetric properties — Their M-theory lift — S F -strings
  - Matrix theory with S-instantons — S/T-duality on S-brane actions
  - Noncommutative S-branes — Non-Abelian S-brane actions etc....
- $\boldsymbol{g_s \neq 0}$?