# Wrapped membrane について

based on collaboration with

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partly on hep-th/0107103

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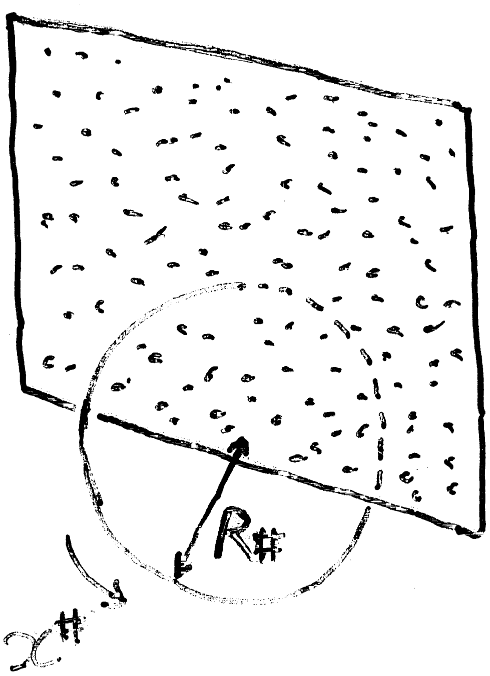
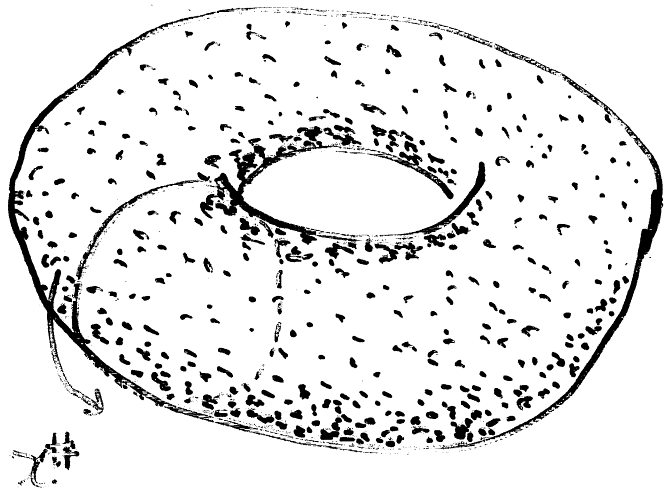
(RIKEN)

# Membranes in 11 dimension

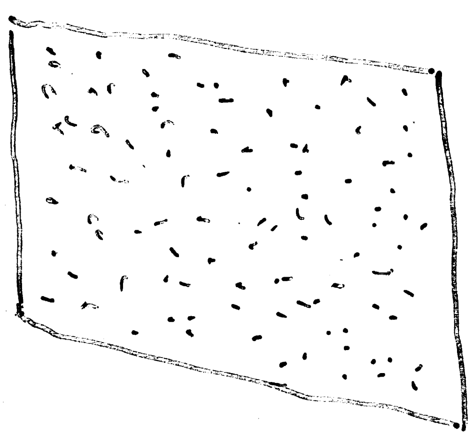
unwrapped membrane



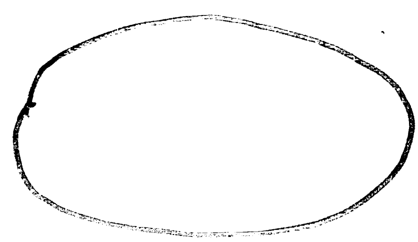

wrapped membrane




$\Downarrow \quad R_\# \to 0$



D2 - brane



string

Membrane action in the light-cone gauge
$( T = 1 )$

$$S^{lc} = \int dt\, d^2\vec{\sigma} \left[ \frac{1}{2} (D_t X^A)^2 - \frac{1}{4} (\{X^A, X^B\})^2 \right]$$

$A = 1, \cdots, D-2$

$$X^{\pm} = \frac{1}{\sqrt{2}} ( X^{D-1} \pm X^0 )$$

$$D_t X^A = \partial_t X^A + \{ A_t, X^A \}$$

$$\{A, B\} = \varepsilon^{rs} \partial_r A\, \partial_s B \qquad (r, s = 1, 2)$$

$S^{lc}$ is invariant under
the residual symmetry,
area preserving doffeomorphism ;

$$\delta_\Lambda X^A = \{\Lambda, X^A\} \qquad \Lambda = \Lambda(t)$$

$$\delta_\Lambda A_t = D_t \Lambda$$

Perturbation theory of membrane ?

( at least as an effective theory )

$(\{X^A, X^B\})^2 \sim (\partial_1 X^A \partial_2 X^B)^2$

.... lack of $(\partial_r X^A)^2$ !!

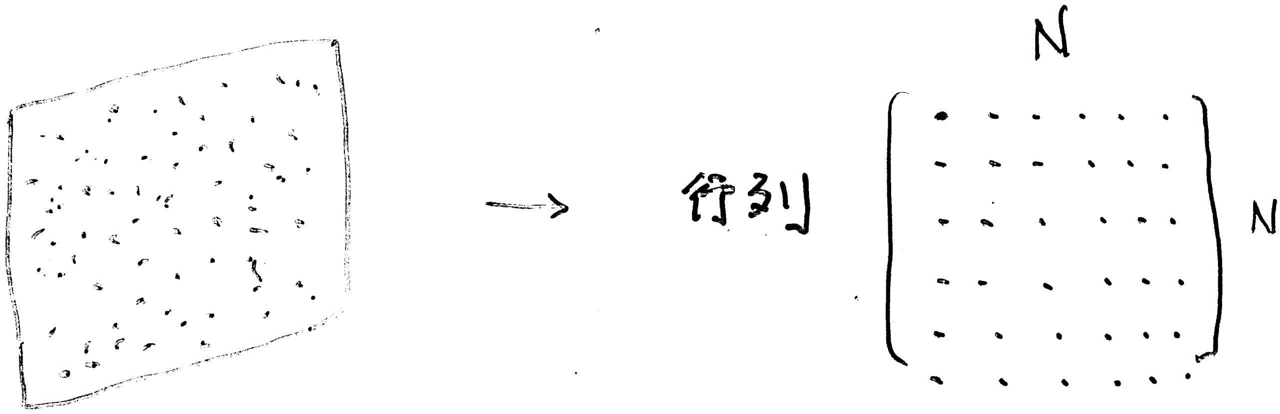
$$\Rightarrow \quad \langle X^A(t,\vec{\sigma})\, X^B(t',\vec{\sigma}')\rangle_{\text{free}}$$

$$= \delta^{AB}\, D(t-t')\, \underline{\delta^2(\vec{\sigma}-\vec{\sigma}')}$$

It needs regularization.

Familiar example

is matrix regularization




area preserving diffeo. $\rightarrow$ U(N) gauge symmetry

$$S_{MQ} = \int dt \, \frac{1}{N} \mathrm{Tr}\left[ \frac{1}{2}(D_t X^A)^2 - \left(\frac{N}{2\pi}\right)^2 \frac{1}{4} \left( i[X^A, X^B] \right)^2 \right]$$

de Wit, Hoppe & Nicolai

It gives a definition of unwrapped membrane.

Let us wrap the $\sigma^2$-direction $(\sigma^2 \approx \sigma^2 + 2\pi)$

once around $x^{D-2} \approx x^{D-2} + 2\pi R$

$$\Leftrightarrow \quad X^{D-2}(t, \sigma^1, \sigma^2 + 2\pi)$$

$$= X^{D-2}(t, \sigma^1, \sigma^2) + 2\pi R$$

If we write $X^{D-2}$ as

$$X^{D-2}(t, \sigma^1, \sigma^2) = R(\sigma^2 - A_1(t, \sigma^1, \sigma^2))$$

then

$$A_1(t, \sigma^1, \sigma^2 + 2\pi) = A_1(t, \sigma^1, \sigma^2)$$

$$\cdot \; -\frac{1}{4}(\{X^A, X^B\})^2$$

$$= -\frac{1}{2}(\{X^{D-2}, X^I\})^2 - \frac{1}{4}(\{X^I, X^J\})^2$$

$$(I, J = 1, \cdots, D-3)$$

$$\{X^{D-2}, X^I\} = R\,(\underbrace{\{\sigma^2, X^I\}}_{\substack{= \varepsilon^{rs}\partial_r\sigma^2\partial_s X^I \\ = -\partial_1 X^I}} - \{A_1, X^I\})$$

$$= -R(\underbrace{\partial_1 X^I + \{A_1, X^I\}}_{\equiv D_1 X^I})$$

$$= -\frac{1}{2}(D_1 X^I)^2 - \frac{1}{4}(\{X^I, X^J\})^2$$

$\Rightarrow$ kinetic term along $\sigma^1$-direction !

$$S^{l.c.}_{WM} = \int dt\, d^2\vec{\sigma} \Big[ -\frac{1}{2}(D_t X^I)^2 - \frac{1}{2}(D_1 X^I)^2 - \frac{1}{4}(\{X^I, X^J\})^2 + \frac{1}{2}(F_{01})^2 \Big]$$

$$\Big( F_{01} \equiv \partial_0 A_1 - \partial_1 A_0 + \{A_0, A_1\} \Big)$$

It still lacks $(\partial_2 X^I)^2$.

But, now we have $(\partial_1 X^I)^2$ in addition to $(\partial_t X^I)^2$

$\Rightarrow$ In terms of matrices, we are inclined to regularize the wrapped membrane due to a 2-dimensional field theory.

Sekino & Yoneya

packed wrapped membrane variables into the matrix variables on 2-dimension!

Plan of this talk



1. Introduction

2. Interest from critical dimension for membranes

3. Sekino - Yoneya procedure for regularizing wrapped membranes

( We can derive S-Y proc. by using D-brains )

4. Lorentz invariance of short distance theory

( ~ matrix string theory )

$$[L^{I-}, L^{J-}] = 0$$

## 2. Interest from critical dimension for membranes

### Anomaly

can arise only when all the covariant metric $\left( \begin{array}{l} \text{induced metric} \quad \partial_a X^M \partial_b X_M \\ \text{another one (hep-th/0109103)} \end{array} \right)$

are degenerate $\left( \det \partial_a X^M \partial_b X_M \Big|_{B.G.} = 0 , \cdots \right)$

on the background around which perturbation is carried out.

Ishibashi & M.H.

Example ⋯ string theory

① $$S_0 = \int d^2\sigma \left[ \frac{1}{2\lambda^0} (\partial_0 X^\mu - \lambda^1 \partial_1 X^\mu)^2 - \frac{\lambda^0}{2} (\partial_1 X^\mu)^2 \right.$$

$$\mu = 0, 1, \cdots, D-1$$

$$\Leftrightarrow \quad S_P = \int d^2\sigma \left[ -\frac{1}{2} \sqrt{-g}\, g^{ab}\, \partial_a X^\mu \partial_b X_\mu \right]$$

with $$\sqrt{-g}\, g^{ab} = \begin{pmatrix} -\frac{1}{\lambda^0} & \frac{\lambda^1}{\lambda^0} \\ \frac{\lambda^1}{\lambda^0} & \frac{(\lambda^1)^2}{\lambda^0} - \lambda^0 \end{pmatrix}$$

Gauge symmetry is 2-dim. diffeomorphism (no Weyl invariance)

Taking the gauge : $\begin{cases} \lambda^0 = 1 \\ \lambda^1 = 0 \end{cases}$

$$S_0 = \int d^2\sigma \, \frac{1}{2} \left( (\partial_0 X^\mu)^2 - (\partial_1 X^\mu)^2 \right)$$

$\Rightarrow$ 2-dimensional diffeomorphism is anomolous except for $D = 26$.

$$S_{NG} = \int d^2\sigma \, \underbrace{\left[ - \sqrt{- \det(\partial_a X^\mu \partial_b X_\mu)} \right]}_{= \mathcal{L}}$$

$$= \int dt \; L$$

$$P_\mu \equiv \frac{\partial L}{\partial(\partial_t X^\mu)}$$

$$\Rightarrow \quad \phi_0 \equiv \frac{1}{2} \left( P^\mu P_\mu + (\partial_\sigma X^\mu)^2 \right) \approx 0$$

$$\phi_1 \equiv P_\mu \partial_\sigma X^\mu \approx 0$$

$$H \equiv \int d\sigma \, [ \, \underbrace{P_\mu \partial_t X^\mu - \mathcal{L}}_{=0}$$

$$+ \lambda^0 \phi_0 + \lambda^1 \phi_1 \, ]$$

$$= \int d\sigma \, [ \, \lambda^0 \phi_0 + \lambda^1 \phi_1 \, ]$$

$$S \equiv \int d^2\sigma \, [ \, P_\mu \partial_t X^\mu \, ) - \int dt \, H$$

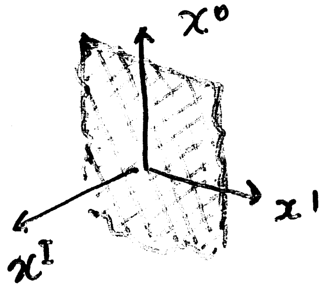
$$= \int d^2\sigma \, [ \, P_\mu \partial_t X^\mu$$

$$- \frac{\lambda^0}{2} \left( (P_\mu)^2 + (\partial_\sigma X^\mu)^2 \right)$$

$$- \lambda^1 P_\mu \partial_\sigma X^\mu \, ] \qquad + \frac{\lambda^0}{2} (\partial_t X^\mu - \lambda^1 \partial_\sigma X^\mu$$

$$= \int d^2\sigma \, \left[ - \frac{\lambda^0}{2} \left( P^\mu - \frac{1}{\lambda^0} (\partial_t X^\mu - \lambda^1 \partial_\sigma X^\mu) \right)^2 - \frac{\lambda^0}{2} (\partial_\sigma X^\mu)^2 \right.$$

(2) $S$ around
$$\begin{cases} X^0_{(0)} = R\sigma^0 \\ X^1_{(0)} = R\sigma^1 \\ X^I_{(0)} = 0 \end{cases} \quad \cdots\cdots (*)$$




$$\begin{cases} X^0 = R(\sigma^0 + y^0) \\ X^1 = R(\sigma^1 + y^1) \\ X^I = R y^I \end{cases}$$
expansion in $\frac{\alpha'^{1/2}}{R}$

We can find the counterterm $\mathcal{O}$ which is well-defined around $(*)$ to cancel $c$.

$$S = S_0 + \beta \int d^2\sigma\, \mathcal{O}$$

... invariant under the modified diffeo.

$$\Rightarrow T_{--} = T^{(0)}_{--} + \beta T^{(0)}_{--}$$

$$T_{--}(\sigma^-)\, T_{--}(\sigma'^-) \sim \frac{c/2}{(\sigma^- - \sigma'^-)^4} + \cdots$$

$$c = D - 26 + 12\beta$$
$$= 0 \quad \text{for } \beta = \frac{26-D}{12}$$

No critical dimension !

.... Quantum consistency of stringy objects appearing in 2, 3, 4-dim. gauge theory

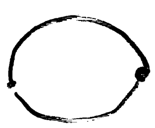
Polchinski & Strominger

$$T_{--} = -\frac{1}{2\alpha'^2}\,\partial_- X\cdot\partial_- X + \frac{\beta}{2}\,\frac{\partial_+ X\cdot\partial_-^3 X}{\partial_+ X\cdot\partial_- X}$$

$$= \frac{R^2}{\alpha'^2}\,\frac{1}{2\sqrt{2}}\,\partial_-(y^0+y^1) - \frac{R^2}{2\alpha'^2}\,\partial_- y\cdot\partial_- y$$

$$+ \beta\,\frac{1}{2\sqrt{2}}\,\partial_-^3(y^0-y^1) + O\left(\frac{\alpha'^2}{R^2}\right)$$

$$\langle y^\mu(\sigma^-)\, y^\nu(\sigma'^-)\rangle = -\frac{\alpha'^2}{R^2}\,\eta^{\mu\nu}\ln(\sigma^- - \sigma'^-)$$

 $\to$ いつもの

 $\propto \beta$

① … fluctuation around "null" background

$$X^\mu_{(0)} = 0$$

$$\Rightarrow \quad \det\left(\partial_a X^\mu_{(0)} \partial_b X_{(0)\mu}\right) = 0$$

② … fluctuation around "static" background

$$\begin{cases} X^0_{(0)} = \sigma^0 \\ X^1_{(0)} = \sigma^1 \end{cases}$$

$$\Rightarrow \quad \det\left(\partial_a X^\mu_{(0)} \partial_b X_{(0)\mu}\right)$$

$$= \det\left(\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

$$\neq 0$$

$$\theta = \frac{1}{4\pi} \partial_a \phi \, \partial^a \phi$$

with $\quad \phi = \ln\left[\det \partial_a X^\mu \partial_b X_\mu\right]$

① $\Rightarrow$ $\theta$ cannot be defined

$\Rightarrow$ anomaly $\propto$ (D-26)

② $\Rightarrow$ $\theta$ is well defined perturbatively

$\Rightarrow$ no anomaly

Explicit calculation shows that

the perturbation theory around

the static membrane $\begin{cases} X^0_{(0)} = \sigma^0 \\ X^1_{(0)} = \sigma^1 \rightarrow (\partial_2 X^I)^2 \\ X^2_{(0)} = \sigma^2 \rightarrow (\partial_1 X^I)^2 \end{cases}$

do not have anomaly for 3 dim. world volume

diffeomorphism.

$X^2_{(0)} = \sigma^2 \Rightarrow$ wrapped membrane ✡

with another choice of gauge $\rightarrow \underset{R_2 \to 0}{\Rightarrow}$ Polyakov string in covariant gauge

$X^0_{(0)} = \sigma^0 \Rightarrow$ membrane wrapped around time-like direction

$\underset{R_0 \to 0}{\Rightarrow}$ Schild-string

Both have critical dimension = 26

If we can treat ✡

without dimensional reduction perturbatively,

we may (be able to) ask the existence of critical dimension of membranes.

# 3. Sekino - Yoneya procedure for regularising wrapped membrane

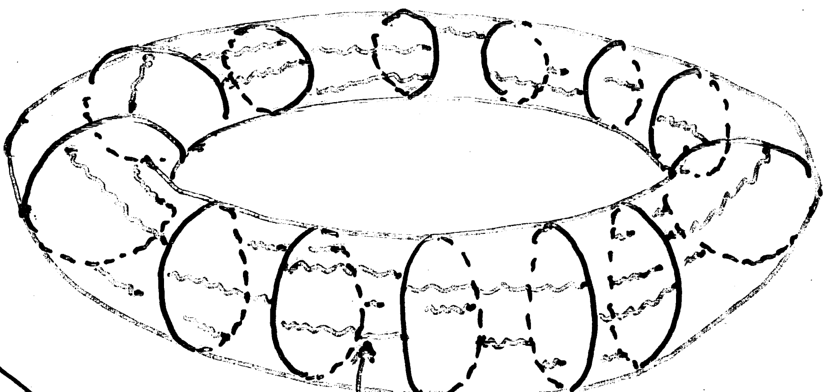
$$S^{lc}_{WM} = \int dt\, d^2\vec{\sigma} \Big[ \frac{1}{2} (D_t X^I)^2 - \frac{1}{2} (D_1 X^I)^2 - \frac{1}{4} (\{X^I, X^J\})^2 + \frac{1}{2} (F_{01})^2 \Big]$$

( If we ignore $A_1$ )

Wrapped membrane can be viewed as

↑

e.x. ~~torus topology~~ for spatial part



$$\boxed{\frac{1}{4} (\{X^I, X^J\})^2}$$

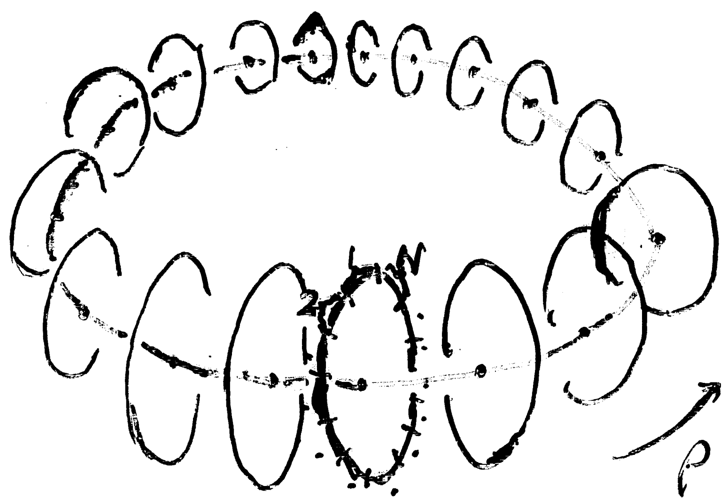
$x^{10}$ & $\sigma^2$ discretized

Ishibashi & M.H.

Sekino - Yoneya procedure :

For a given (large) $N$,

discretize $\rho \equiv \sigma^2$ - direction $\left(\begin{array}{l}\text{wrapped}\\ \text{direction}\end{array}\right)$
into $N$ points,
and divide $\sigma \equiv \sigma^1$ - coordinate
into $N$ - segment.




$$\Rightarrow \quad X^I(\underset{\uparrow}{\tau}, \sigma, \rho) = \underset{\uparrow}{\sum_n} e^{-in\rho} \underline{X_n^I}(\tau, \sigma)$$

$\left(\begin{array}{l}\text{suppressed}\\ \text{below}\end{array}\right)$ # (KK mode) = $N$

Prepare $N \times N$ matrices $x^I(\tau, \theta)$

$$\theta \approx \theta + 2\pi$$

Map :

$$X_n^I(\sigma) = x^{I\,k}{}_{l}(\theta)$$

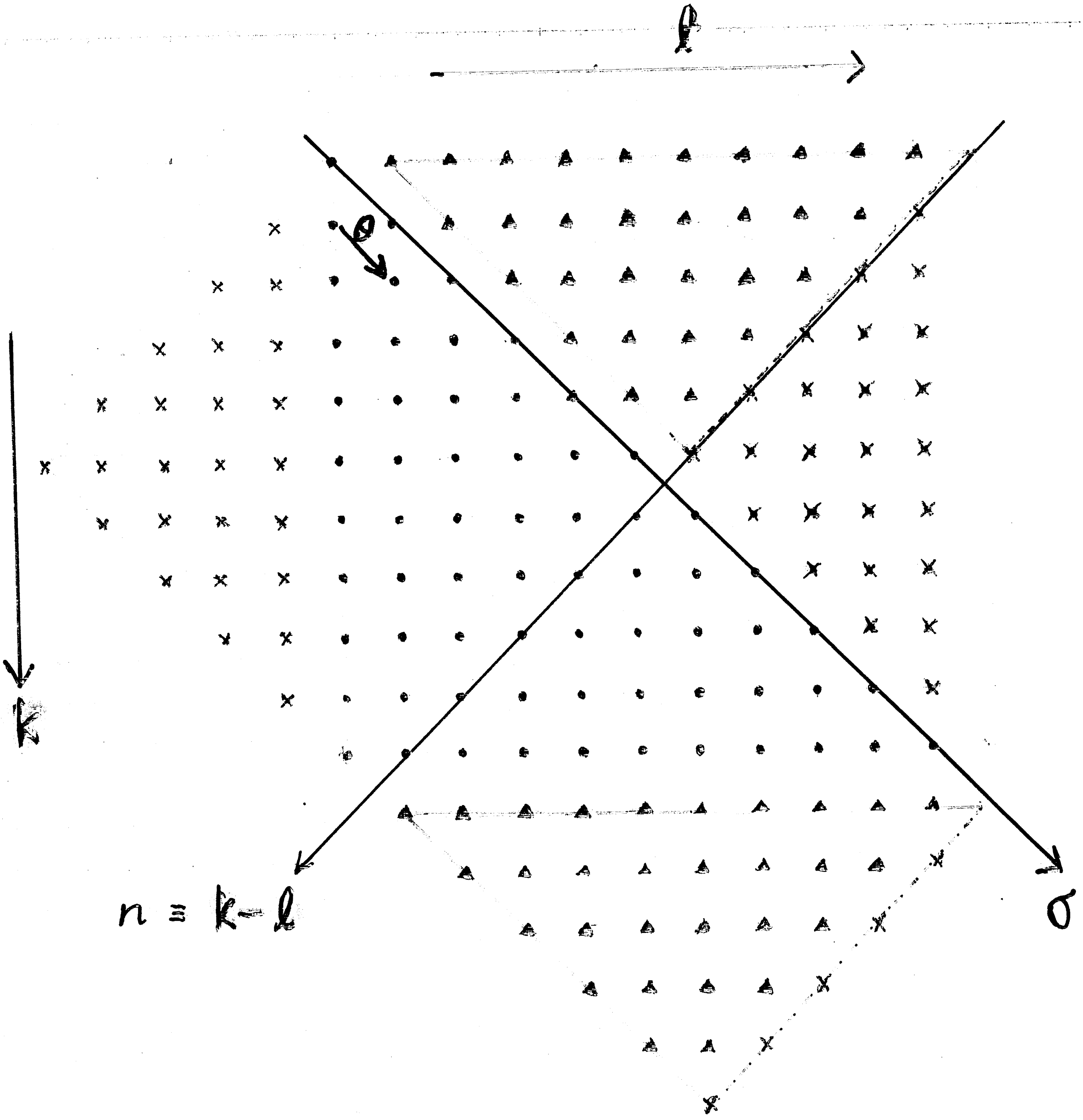
with

$$n = k - l$$

$$\sigma = \frac{(k-1)+(l-1)}{N}\pi + \frac{\theta}{N}$$

Sekino-Yoneya mapping rule

$N=11$



$$S^{reg}_{WM} = \int dt \int_0^{2\pi} \frac{d\theta}{2\pi} \frac{1}{N} \mathrm{tr}\Big[ -\frac{1}{2}(D_t x^I)^2 - \frac{N^2}{2}(D_\theta x^I)^2$$
$$- \frac{N^2}{(2\pi)^2 R^3}\frac{1}{4}(i[x^I, x^J])^2$$
$$+ \frac{1}{2}(f_{t\theta})^2 \Big]$$

$$f_{t\theta} \equiv \partial_t a_\theta - N \partial_\theta a_t - i\frac{N}{2\pi R^{3/2}}[a_t, a_\theta]$$

$$\underset{(\theta)}{D_t x^I} \equiv \underset{(\theta)}{\partial_t x^I} - i\frac{N}{2\pi R^{3/2}} [\underset{(\theta)}{a_t}, x^I]$$

Using

$$\tau \equiv Nt, \qquad a_\tau \equiv a_t$$

$$S^{reg}_{WM} = S_{MS}$$
$$= \int d\tau \int_0^{2\pi} \frac{d\theta}{2\pi} \mathrm{tr}\Big[ -\frac{1}{2}(D_\tau x^I)^2 - \frac{1}{2}(D_\theta x^I)^2$$
$$- \frac{1}{(2\pi)^2 R^3}(i[x^I, x^J])^2$$
$$+ \frac{1}{2}(f_{\tau\theta})^2 \Big]$$

---- action in matrix string theory

with

$$x^{I\,k}{}_{l}(\theta + 2\pi) = x^{I\,k+1}{}_{l+1}(\theta)$$

One can "derive" Sekino-Yoneya procedure from D-branes :

$MN$ # of D0 $\longrightarrow$ D2-brane

wrap $\rho$ around $x^{10}$

$\updownarrow$

$\langle x^{D-2}(\tau)\rangle$

$= R\,\mathrm{diag}\left(0, \frac{2\pi}{MN}, \cdots, \frac{2\pi}{MN}(MN-1)\right)$

$\longrightarrow$ $N$ D1-branes

with $x^{I\,k}{}_{\ell}(\theta+2\pi) = x^{I\,k+1}{}_{\ell+1}(\theta)$

$\theta \in \{0, \frac{2\pi}{M}, \cdots, \frac{2\pi}{M}(M-1)\}$

Note :

Use of $\tau$

$\Rightarrow$ action in the canonical form of the matrix string theory.

$t = \frac{1}{N}\tau$ is directly associated

( More precisely $\sigma^0 = \frac{1}{R} t = \frac{1}{NR}\tau$ )

with regularized membranes.

For $t \sim O(1)_{2\pi}$, $\tau$ needs $O(N)$!

So, the membrane dynamics is described by the infrared dynamics of matrix string action

It has t'Hooft coupling : $g^{t}_{mm} = \left(\frac{N}{R}\right)^{3/2}$

$$S^{reg.}_{WM} = \int dt \int_0^{2\pi} \frac{d\theta}{2\pi} \frac{1}{N} \mathrm{tr}\Big[ -\frac{1}{2}(\partial_t x^I)^2 - \frac{N^2}{2}(D_\theta x^I)^2$$

$$- \frac{N^2}{(2\pi)^2 R^3} \frac{1}{4} \left( i[x^I, x^J] \right)^2$$

$$+ \frac{1}{2} (f_{t\theta})^2 \Big]$$

gives a definition of wrapped membranes

Using (Sekino - Yoneya)

$$\int_0^{2\pi} \frac{d\theta}{2\pi} \frac{1}{N} \mathrm{tr}\left[ \phi_{(1)}(\theta) \cdots \phi_{(p)}(\theta) \right]$$

$$= \frac{1}{N} \sum_{\rho} \int_0^{2\pi} \frac{d\sigma}{2\pi} \boxed{ e^{\frac{\partial \pi}{N} \sum_{1 \le j < k \le p} \left( \partial_{\sigma(j)} \partial_{\rho(k)} - \partial_{\rho(j)} \partial_{\sigma(k)} \right)} }$$

$$\cdot \Phi_{(1)}(\sigma_{(1)}, \rho_{(1)}) \cdots \Phi_{(p)}(\sigma_{(p)}, \rho_{(p)}) \Big|_{\substack{\sigma(j) = \sigma \\ \rho(j) = \rho}}$$

( $\partial_\rho$ : difference operator )

$$S^{reg}_{WM} = \int dt \int_0^{2\pi} \frac{d\sigma}{2\pi} \frac{1}{N} \sum_{\rho} \Big[ -\frac{1}{2}(D_t X^I)^2 - \frac{1}{2}(D_\sigma X^I)^2$$

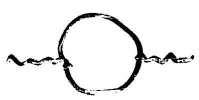
$$- \frac{N^2}{(2\pi)^2 R^3} \frac{1}{4} \left( i[X^I, X^J] \right)^2$$

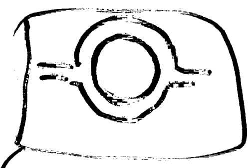
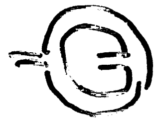
$$+ \frac{1}{2} (F_{t\sigma})^2 \Big]$$

$$\begin{cases} F_{t\sigma} \equiv \partial_t A_\sigma - \partial_\sigma A_t - i \frac{N}{2\pi R^{3/2}} [A_t, A_\sigma]_* \\ D_a X^I \equiv \partial_a X^I - i \frac{N}{2\pi R^{3/2}} [A_a, X^I]_* \end{cases}$$

$$\langle Y^I(t,\sigma,\rho)\, Y^J(t',\sigma',\rho')\rangle$$

$$= \delta^{IJ}\, D(t-t',\, \sigma-\sigma')\, \underline{\delta_{\rho\rho'}}$$



$=$  $+$ 

 $\Rightarrow \quad \sum_{\rho} \delta_{\rho\rho} = N$

$$N \times \left(\frac{N}{2\pi R^{3/2}}\right)^2 \sim \left[\left(\frac{N}{R}\right)^{3/2}\right]^2$$

# 4. Lorentz invariance of short distance theory

It is difficult to access to the infrared aspects directly.

The short distance $\tau = O(1)$

$\sim$ matrix string theory

$\xleftrightarrow{\text{dual}}$ M(atrix) theory

with total $p^+ = \frac{N}{R_\#}$ : fixed

$\leftrightarrow$ old extra direction

$$l\, g_{MS} = \frac{l}{2\pi} \frac{1}{R^{3/2}} \ll 1 \qquad \text{for } R \gg 1$$

$$l \sim O(2\pi)$$

We require the existence of such a theory and consistency with Lorentz invariance as possible at each order of $g_{MS}$.

For that purpose,

we would like to change the gauge fixing condition

of the membrane action.

light-cone gauge : $p^+ = \text{const}$

especially along $\rho$

... not suitable to the picture :

membrane ~ independent strings

lying along $\rho$

interating each other

$$\boxed{\text{New gauge fixing :} \quad \partial_\sigma P^+ = 0 \qquad p^+ = p^+(\rho) \qquad A_1 = 0}$$

c.f.) In the light-cone gauge

$p^+ = \text{const}$, $X^+ = t$

$\Rightarrow$ $\partial_\sigma A_1 = 0$ is possible

$A_1(\rho)$ cannot be removed

by area preserving diffeomorphism.

$\delta_\varepsilon A_1 = \partial_0 \Lambda + \{\Lambda, A_1\}$, $\Lambda(\sigma,\rho) = \sum_n e^{in\sigma} \Lambda_n(\rho)$

$\delta_\varepsilon A_{1,n} = \Lambda_n(\rho) + \cdots$

Independent dynamical variables :

$$X^I(\sigma,\rho)\ ,\quad P^I(\sigma,\rho)\ ,$$

$$x^-(\rho)\ ,\quad p^+(\rho)\ ,$$

$$(+\ p^-\quad \text{at off-shell})$$

$$X^-(\sigma,\rho) = \sum_n e^{-in\sigma} X^-_m(\rho)$$

$$X^-_{m\neq 0}(\sigma) \quad \Leftarrow \quad \phi_1 = P^+ \underline{\partial_\sigma X^-} + P^I \partial_\sigma X^I \approx 0$$

$$\Pi_{A_1}(\sigma,\rho) \left( \equiv \frac{\partial \mathcal{L}}{\partial(\partial_0 A_1(\sigma,\rho))} \right)$$

$$\Leftarrow \quad \phi_2 = P^+ \partial_2 X^- + P^I \partial_2 X^I - \underline{\Pi_{A_1}} \approx 0$$

$$P^- \Leftarrow \phi_0 = p^+(\rho)\, \underline{P^-} + \frac{1}{2}(P^I)^2 + \frac{1}{2R^2}(\Pi_{A_1})^2 + \frac{1}{4}\left(\{X^I, X^J\}\right)^2 + \frac{R^2}{2}(D_1 X^I)^2$$

$$\approx 0$$

$$p = \frac{2\pi}{N} m \qquad (m \simeq m+N)$$

At the leading order,

Lorentz generator $L^{MN}$ $(M, N \neq D-2)$
wrapped direction

consists of the sum of

Lorentz generator $l^{MN}_{(m)}$

of each string at the site $m$;

$$L^{MN} = \sum_m l^{MN}_{(m)}$$

$$[L^{-I}, L^{-J}]$$

$$= \sum_m [l^{-I}_{(m)}, l^{-J}_{(m)}]$$

$$= \sum_m \left[ -\frac{1}{(p^+_{(m)})^2} \sum_{r=1}^{\infty} \Delta_r (\alpha^I_{(m)-r} \alpha^J_{(m)r} - \alpha^J_{(m)-r} \alpha^I_{(m)r}) \right]$$

(on-shell)

with

$$\Delta_r = r\frac{26-(D-1)}{12} + \frac{1}{r}\left(\frac{(D-1)-26}{12} + 2(1-a)\right)$$

$\Rightarrow$ $D-1 = 26$ is critical dimension

fate at the next order ??

MN 個の D0-brane で transverse coordinate として

$$x^I_{ab}(t) = x^I_{a-1,\,b-1}(t) \qquad I = 1, \cdots, D-2$$

$$x^{D-1}_{ab}(t) = x^{D-1}_{a-1,\,b-1}(t) + 2\pi R\,\delta_{ab}$$

に従うものを用意。

$a, b = 0, \cdots, M-1 \leftrightarrow$ N×N submatrix を区別

$M \to \infty$ で 半径 R の circle 上の D0-brane を表す。

$$x^{D-1}(t) = -MR\,Q(MN) + \hat{x}^{D-1}(t)$$

$$\hat{x}^{D-1}_{ab}(t) = \hat{x}^{D-1}_{a-1,\,b-1}(t)$$

$$Q(MN) = \mathrm{diag}\left(0, -\frac{2\pi}{MN}, \cdots, -\frac{2\pi}{MN}\times N, \cdots\cdots, -\frac{2\pi}{MN}(MN-1)\right)$$

dual circle $\tilde{x} \approx \tilde{x} + 2\pi\tilde{R}$ 上の

$$\left( \tilde{R} = \frac{1}{R} \right)$$

$N\times N$ matrix $\tilde{X}^I(t, \tilde{x})$, $\tilde{A}_{\tilde{x}}(t, \tilde{x})$

を

周期的

$$\tilde{X}^I(t, \tilde{x}) \equiv \sum_{a=0}^{M-1} e^{-ia\tilde{x}/\tilde{R}} \, x^I_{a0}(t)$$

$$\tilde{A}_{\tilde{x}}(t, \tilde{x}) \equiv \sum_{a=0}^{M-1} e^{-ia\tilde{x}/\tilde{R}} \, \frac{1}{2\pi} x^{D-1}_{a0}(t)$$

と準備.

$$x^{D-1}_{00}(t) = \underline{-RQ_{(N)}} + \hat{x}^{D-1}_{00}(t)$$

$$\Rightarrow \quad \tilde{A}_{\tilde{x}}(t, \tilde{x}) = \underbrace{-\frac{1}{2\pi\tilde{R}} Q_{(N)}}_{\text{background}} + \cdots$$

background を取り除くように

gauge 変換:

$$A_{\tilde{x}} = e^{i\frac{\tilde{x}}{2\pi\tilde{R}}Q_{(N)}} \left( i\partial_{\tilde{x}} + \tilde{A}_{\tilde{x}} \right) e^{-i\frac{\tilde{x}}{2\pi\tilde{R}}Q_{(N)}}$$

$$X^I = e^{i\frac{\tilde{x}}{2\pi\tilde{R}}Q_{(N)}} \tilde{X}^I e^{-i\frac{\tilde{x}}{2\pi\tilde{R}}Q_{(N)}}$$

$$\theta \equiv \frac{\tilde{x}}{\tilde{R}} \quad \Rightarrow \quad \theta \approx \theta + 2\pi$$

$$A_\theta \equiv \tilde{R} A_{\tilde{x}}$$

ゲージ変換は dual circle上 periodic でない

から.

$$A_\theta(\theta + 2\pi) = e^{iQ(N)} A_\theta(\theta) e^{-iQ(N)}$$

$$X^I(\theta + 2\pi) = e^{iQ(N)} X^I(\theta) e^{-iQ(N)}$$