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Moyal Formulation of Open String Field Theory

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based on

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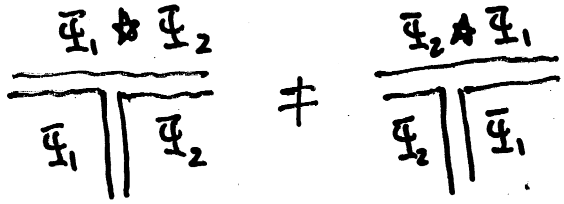
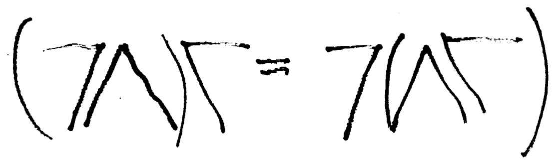
hep-th/0211131, 0212XXX

§1. Introduction

Witten's Open string field theory

based on Noncommutative & associative product



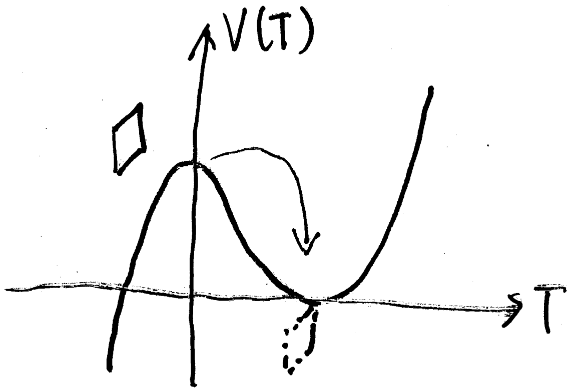


$\star$ - product is similar to matrix multiplication

$\Downarrow$

$\boxed{\text{Noncommutative geometry}}$

String field theory as $\boxed{\text{off-shell}}$ formulation



Tachyon condensation

brane creation/annihilation

$\wr\wr$

off-shell process

- Computation of tension (Numerical) 99.99.. %

Noncommutative soliton (projector)

$\| ?$

D- brane

Problems in OSFT

① Subtlety at the midpoint

Witten's star product (in half-string language)

$$(\hat{\Psi}_1 * \hat{\Psi}_2)(l, r) = \int Dt \; \hat{\Psi}_1(l, t)\, \hat{\Psi}_2(t, r)$$

$$X(\sigma) = \begin{cases} l(\sigma) & 0 \le \sigma \le \frac{\pi}{2} \\ r(\pi - \sigma) & \frac{\pi}{2} \le \sigma \le \pi \end{cases}$$

$l(\sigma)$ $r(\sigma)$

$X(\sigma)$

This splitting is singular at the midpoint

② Level truncation

$\Psi \to \Psi_N$: cut-off at level N

$$S(\Psi) \to S_N(\Psi_N) = \frac{1}{2}\int' \Psi_N *' Q \Psi_N + \frac{1}{3}\int' \Psi_N *' \Psi_N *' \Psi_N$$

We need cut-off for $\int'$, $*'$, Q ...

Level truncation is not well-defined at the Lagrangian level

conformal symmetry? unitarity?

$\Rightarrow$ We need regularization at the Lagrangian level in order to perform analytic computation.

Plan

§1 Introduction & Motivation

§2. Moyal Formulation of Witten's star product (Bars)

§3. Anomaly in associativity & a proposal of regularization (BM1)

§4. Algebra of string states (BM2)
- Projector
- Neumann coefficient

§5. Computing string diagrams (BKM1)

§6 Tachyon condensation & classical solutions (BKM2)

§7. Discussion

③ Naive manipulation by using $N \to \infty$ limit at the Lagrangian level leads to $\begin{pmatrix}\text{meaningles} \\ \text{conflicting}\end{pmatrix}$ results

<u>example</u>

a) Purely cubic theory

$$S_{\text{Witten}}[\Psi] = \frac{1}{2}\int \Psi * Q\Psi + \frac{1}{3}\int \Psi^3$$

$$\downarrow \quad \Psi = Q_L I + \Psi'$$

$$S_{\text{cubic}}[\Psi'] = \frac{1}{3}\int (\Psi')^3$$ Brane annihilation

but $S_{\text{Witten}}[Q_L I] = 0$ No tension!

b) VSFT

Hata-Kawano solution

= insertion of vertex operator at midpoint ⇒ Wrong tension

Okawa's solution

= (marginal) deformation of CFT ⇒ Correct tension

BUT Both are solution of VSFT e.o.m.

Is VSFT well-defined?

§2 Moyal Formulation of Witten's star product

(Bars hep-th/0106157)

From integration to Moyal Product

◎ Two variable case

$\hat{\psi}_i(l, r) \quad i = 1, 2.$

$$(\hat{\psi}_1 \star \hat{\psi}_2)(l, r) = \int_{-\infty}^{\infty} dt \; \hat{\psi}_1(l, t)\, \hat{\psi}_2(t, r)$$

$\Downarrow$ Fourier transformation

$$\mathcal{F}(\hat{\psi})(x, p) = \int_{-\infty}^{\infty} \psi\left(\frac{x+y}{2}, \frac{x-y}{2}\right) e^{-ipy}\, dy$$

$$[\mathcal{F}(\hat{\psi}_1) * \mathcal{F}(\hat{\psi}_2)](x, p) = \mathcal{F}(\hat{\psi}_1 \star \hat{\psi}_2)(x, p)$$

$$(A_1 * A_2)(x, p) = e^{\frac{i}{2}(\partial_x \partial_{p'} - \partial_p \partial_{x'})} A_1(x, p)\, A_2(x', p')\Big|_{\substack{x' = x \\ p' = p}}$$

String field theory

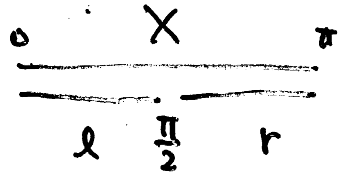
$$(\hat{\varphi}_1 \star \hat{\varphi}_2)(l, r) = \int \underline{Dt}\; \hat{\varphi}_1(l, t)\, \hat{\varphi}_2(t, r)$$

$\Uparrow$ Path integral

X^μ : map $[0, \pi] \to \mathbb{R}^d$

l^μ, r^μ : map $[0, \frac{\pi}{2}] \to \mathbb{R}^d$

splitting variable X^μ



Path integral must be specified more concretely in terms of mode expansion.

$X(\sigma)$: Neumann boundary condition at $\sigma = 0, \pi$

$$X(\sigma) = x_0 + \sqrt{2} \sum_{n>0} x_n \cos(n\sigma)$$

$l(\sigma), r(\sigma)$: Neumann at $\sigma = 0$

boundary condition at $\sigma = \frac{\pi}{2}$??

Dirichlet ?

Neumann ?

Dirichlet at $\sigma = \frac{\pi}{2}$

$$l^\mu\left(\tfrac{\pi}{2}\right) = r^\mu\left(\tfrac{\pi}{2}\right) = \bar{x}^\mu$$

$$l^\mu(\sigma) = \bar{x}^\mu + \sqrt{2}\sum_{o>0} l_o^\mu \cos(o\sigma)$$

$$r^\mu(\sigma) = \bar{x}^\mu + \sqrt{2}\sum_{o>0} r_o^\mu \cos(o\sigma)$$ o : odd number

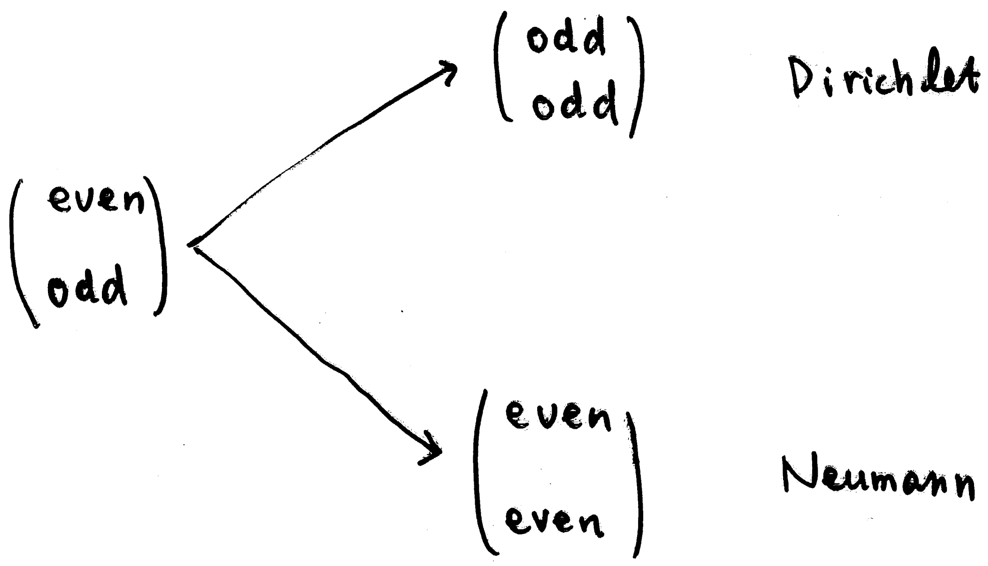
Neumann at $\sigma = \frac{\pi}{2}$

$$l^\mu(\sigma) = l_0^\mu + \sqrt{2}\sum_{e>0} \cos(e\sigma)\, l_e^\mu$$ e : even number

$$r^\mu(\sigma) = r_0^\mu + \sqrt{2}\sum_{e>0} \cos(e\sigma)\, r_e^\mu$$

Original

Split string



$\boxed{\text{We need translation of even} \leftrightarrow \text{odd modes}}$

Translation = Fourier transformation

$$\begin{cases} T_{eo} = \frac{4}{\pi}\int_0^{\pi/2} \cos(e\sigma)\cos(o\sigma)d\sigma = \frac{2 i^{e+o-1}}{\pi}\left(\frac{1}{o+e} - \frac{1}{o-e}\right) \\ R_{oe} = \frac{4}{\pi}\int_0^{\pi/2} \cos(o\sigma)\left(\cos(e\sigma) - i^e\right)d\sigma = \frac{2e\, i^{e+o+1}}{\pi o}\left(\frac{1}{o+e} - \frac{1}{o-e}\right) \\ v_o = \frac{1}{\sqrt{2}} T_{\emptyset o} = \frac{2\sqrt{2}}{\pi}\frac{i^{o-1}}{o} \\ w_e = -\sqrt{2}\, i^e \qquad (i = \sqrt{-1}) \end{cases}$$

$$\boxed{\mathcal{H}^{odd} \underset{R}{\overset{T}{\rightleftarrows}} \mathcal{H}^{even} \qquad v \in \mathcal{H}^{odd},\quad w \in \mathcal{H}^{even}}$$

With the help of (T, R, v, w), one may define

$$A(x_e, p_e) \propto \int dx_o\, e^{-\frac{2i}{\theta}\sum_{e,o>0} p_e T_{eo} x_o} \cdot \psi(\underbrace{\bar{x} + \sum_e w_e x_e}_{x_\emptyset};\ x_e, x_o)$$

$\boxed{\text{Witten's star product} = \text{Moyal product for } A}$

$$(A_1 * A_2)(x_e, p_e) = e^{\frac{i}{2}\partial_\xi \sigma \partial_{\xi'}} A_1(\xi) A_2(\xi')\Big|_{\xi'=\xi}$$

$$\xi = \begin{pmatrix} x_e \\ p_e \end{pmatrix} \qquad \sigma = \begin{pmatrix} 0 & i\theta \\ -i\theta & 0 \end{pmatrix}$$

§3. Anomaly in associativity & Regularization

Bars & M hep-th/0202030

Algebra of (T, R, v, w) $(+\kappa_e, \kappa_o)$

$$
\begin{cases}
TR = RT = 1 & R = \bar{T} + v\bar{w}, \quad R = \kappa_o^{-2}\,\bar{T}\,\kappa_e^{2} \\
Tv = 0, & v = \bar{T}w \\
T\bar{T} = 1, & \bar{T}T = 1 - v\bar{v} \\
\bar{v}v = 1 & \kappa_o = \begin{pmatrix}1\\3\\5\\7\\ \vdots\end{pmatrix} \quad \kappa_e = \begin{pmatrix}2\\4\\6\\8\\ \vdots\end{pmatrix}
\end{cases}
$$

$(\bar{T} \equiv \text{transpose}(T))$

This algebra is <u>not</u> associative!

$$R(Tv) = 0 \quad \longleftrightarrow \quad (RT)v = v$$

$$T(\bar{T}w) = 0 \quad \longleftrightarrow \quad (T\bar{T})w = w$$

$$(\bar{w}T)v = 1 \quad \longleftrightarrow \quad \bar{w}(Tv) = 0$$

$$\boxed{\text{Witten's star product is not well-defined without the introduction of regularization which recover the associativity.}}$$

Deformed (Regularized) Algebra (associative)

$$\begin{cases} TR = RT = 1, \quad R = \bar{T} + v\bar{w}, \quad R = \kappa_o^{-2}\bar{T}\kappa_e^2 \\ \bar{R}R = 1 + w\bar{w}, \quad \bar{T}T = 1 - v\bar{v} \\ T\bar{T} = 1 - \dfrac{w\bar{w}}{1+\bar{w}w}, \quad Tv = \dfrac{w}{1+\bar{w}w}, \quad \bar{v}v = \dfrac{\bar{w}w}{1+\bar{w}w} \\ Rw = v(1+\bar{w}w), \quad R\bar{R} = 1 + v\bar{v}(1+\bar{w}w) \end{cases}$$

$\kappa_e = (\kappa_2, \kappa_4, \cdots, \kappa_{2N})$ $\kappa_o = (\kappa_1, \kappa_3, \cdots \kappa_{2N-1})$

arbitrary spectrum!

This deformed algebra reduces to original one in

$$\boxed{\bar{w}w \to \infty \quad \text{limit}}$$

$\{T, R, v, w\}$ are realized by $N \times N$ matrices & N vectors

$$T_{eo} = \frac{w_e v_o \kappa_o^2}{\kappa_e^2 - \kappa_o^2} \qquad R_{oe} = \frac{w_e v_o \kappa_o^2}{\kappa_e^2 - \kappa_o^2}$$

$$w_e = i^{2-e} \frac{\prod_{o} |\kappa_e^2/\kappa_o^2 - 1|^{1/2}}{\prod_{e'(\neq e)} |\kappa_e^2/\kappa_{e'}^2 - 1|^{1/2}}, \quad v_o = \cdots$$

$\{\kappa_e\}_{e=2,4,\cdots,2N}$ $\{\kappa_o\}_{o=1,3,\cdots 2N-1}$

free parameters

$$\boxed{\begin{array}{l}\underline{\text{Claim}} \\ \text{All manipulations required to compute} \\ \text{string amplitude can be defined for} \\ \text{arbitrary } \{N, \kappa_e, \kappa_o\} \\ \text{at least in } \underline{\underline{\text{Siegel gauge}}}\end{array}}$$

gauge "fixed" description

- 3-string vertex
- propagator
- external states

<u>Note</u>

- κ_e, κ_o, N : not fixed

  we need to put $\kappa_e \sim e$, $\kappa_o \sim o$, $N \to \infty$

  to recover string amplitude

- How to fix them at finite N?

  - deformed version of conformal symmetry?

- Gauge symmetric formulation

  - construction of nilpotent BRST operator?

§4. Algebra of string states

Perturbative vacuum

$$a_n|0\rangle = 0 \qquad n \geq 0 \quad \overset{\text{finite}}{\underset{N}{\Longrightarrow}} \quad \begin{cases} a_o|0\rangle = 0 \\ a_e|0\rangle = 0 & e, o > 0 \\ a_0|0\rangle = 0 \end{cases}$$

$$|0\rangle \overset{\text{Moyal}}{\Longrightarrow} A_0 = N_0 \exp(-\bar{\xi} M_0 \xi)$$

$$\xi = \begin{pmatrix} x_e \\ p_e \end{pmatrix} \qquad M_0 = \begin{pmatrix} \kappa_e & 0 \\ 0 & T\kappa_o^{-1}\bar{T} \end{pmatrix}$$

Generic perturbative (Fock) space

$$\{ a_{-n_1} \cdots a_{-n_\ell} |p\rangle \}$$

$$a_0|p\rangle = p|p\rangle$$

$$\Downarrow$$

$$\boxed{f(\xi)\, e^{-\bar{\xi} M_0 \xi - ip(\bar{w}\xi)}} \qquad w = \begin{pmatrix} w_e \\ 0 \end{pmatrix}$$

$f(\xi)$: polynomial in ξ

- Generating functional for such states

$$A_{N,M,\lambda} \equiv N e^{-\bar{\xi} M \xi - \bar{\lambda}\xi}$$

Algebra of gaussian function under Moyal $*$

Algebra between gaussian functions

$$A_{N_1, M_1, \lambda_1} * A_{N_2, M_2, \lambda_2} = A_{N_{12}, M_{12}, \lambda_{12}}$$

$$\begin{cases} m_{12} = (m_1 + m_2 m_1)(1 + m_2 m_1)^{-1} + (m_2 - m_1 m_2)(1 + m_1 m_2)^{-1} \\ \lambda_{12} = (1 + m_2)(1 + m_1 m_2)^{-1} \lambda_1 + (1 - m_1)(1 + m_2 m_1)^{-1} \lambda_2 \\ N_{12} = \dfrac{N_1 N_2}{\det (1 + m_2 m_1)^{d/2}} \, e^{\frac{1}{4}((\bar{\lambda}_1 + \bar{\lambda}_2)\sigma (m_1 + m_2)^{-1} (\lambda_1 + \lambda_2) - \bar{\lambda}_{12} \sigma m_{12}^{-1} \lambda_{12})} \end{cases}$$

$$m_1 = M_1 \sigma, \quad m_2 = M_2 \sigma, \quad m_{12} = M_{12} \sigma$$

Some properties of algebra

- Unit 1, closed
- Some elements do not have inverse

"Monoid"

Projector

$$A * A = A$$

$$\Leftrightarrow \begin{cases} m^2 = 1 \\ \lambda = 0 \end{cases}$$

always rank 1 projector

Example

Sliver $M_s = m_0 (m_0^2)^{-\frac{1}{2}}$, Butterfly $M_B = \begin{pmatrix} M_c & 0 \\ 0 & M_c^{-1} \end{pmatrix}$

Multiple product

n^{th} product

$$A_{N_1 M_1 \lambda_1} * A_{N_2, M_2, \lambda_2} * \cdots * A_{N_n, M_n, \lambda_n} = A_{N_{12\cdots n} M_{12\cdots n} \lambda_{12\cdots n}}$$

For the simple situation where

$$[m_i, m_j] = 0 \qquad (m_i = M_i \sigma)$$

one can derive the explicit form of product.

$$M_{12\cdots n}\sigma = \frac{J^-_{12\cdots n}}{J^+_{12\cdots n}}, \quad J^{\pm}_{12\cdots n} = \frac{1}{2}\left(\prod_{k=1}^{n}(1+m_k) \pm \prod_{k=1}^{n}(1-m_k)\right)$$

$$\lambda_{12\cdots n} = \sum_{i=1}^{n} \frac{\prod_{k=1}^{i-1}(1-m_k)\prod_{l=i+1}^{n}(1+m_l)}{J^+_{12\cdots n}} \cdot \lambda_i$$

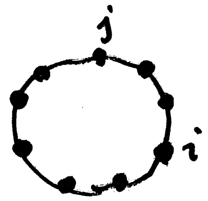
$$N_{12\cdots n} = \frac{N_1 \cdots N_n}{\det(J^+_{12\cdots n})^{d/2}} \exp\left(\frac{1}{4} K_{12\cdots n}\right)$$

$K_{12\cdots n}$ = (Quadratic term of λ)

And

$$\mathrm{Tr}(A_1 * \cdots * A_n) = \frac{N_1 \cdots N_n}{\det(2J^-_{12\cdots n})^{d/2}} \exp\left(\frac{1}{4} Q_{12\cdots n}\right)$$

$$Q_{12\cdots n} = \sum_{i=1}^{n} \bar\lambda_i \sigma \frac{J^{+(i)}_{12\cdots n}}{J^-_{12\cdots n}} \lambda_i + \sum_{i\neq j} \bar\lambda_i \sigma \frac{\prod_{k=j+1}^{i-1}(1+m_k)\prod_{k=i+1}^{j-1}(1-m_k)}{J^-_{12\cdots n}} \lambda_j$$



When $M_i = M_0$ (perturbative vacuum)

$$A_i \longleftrightarrow e^{\sum \mu_n^{(i)} a_n^+} |0\rangle \equiv |i\rangle$$

$$\mathrm{Tr}(A_1 * \cdots * A_N) \Leftrightarrow \langle V_n | 1\rangle \otimes |2\rangle \otimes \cdots \otimes |n\rangle$$

$$\langle V_n| = \langle 0| \otimes \cdots \otimes \langle 0| \exp\left(\sum_{\substack{ij \\ rs}} V_{rs}^{(ij)} \, a_r^{(i)} a_s^{(j)}\right)$$

$V_{rs}^{(ij)}$ are called Neumann coefficient and are essential to give any quantity in SFT

① They are usually derived from CFT indirectly. However MSFT can give their algebraic constructio in terms of well-known matrices T & R

② $V_{rs}^{(ij)}$ satisfy a family of nonlinear relations (Gross-Jevicki)

$$V_{ab}^{[rt]} V_{bc}^{[ts]} = \delta_{rs} \delta_{ac}, \quad V_{ab}^{[rt]} V_{b0}^{[t,s]} = V_{a,0}^{[r,s]}, \cdots$$

In our construction, they are simple consequence of the Moyal algebra.

③ Our regularization gives finite size representation of V. It should be important in analytic and/or numeric computation of SFT.

§5. String perturbation

Witten's action in Siegel gauge

$$S = \int \left(\frac{1}{2} \Phi * c_0 (L_0 - 1) \Phi + \frac{1}{3} \Phi * \Phi * \Phi \right)$$

$\Downarrow$ mapped to

$$\boxed{\begin{aligned} S &= \frac{1}{2} \int d^d \bar{x} \, \mathrm{Tr} \left(A * (L_0 - 1) A \right) + \frac{1}{3} \int d^d \bar{x} \, \mathrm{Tr} \left(A * A * A \right) \\ L_0 &= -\frac{1}{4} a_{ij} \frac{\partial}{\partial \xi^i} \frac{\partial}{\partial \xi^j} + b_{ij} \xi^i \xi^j + c_i \frac{\partial}{\partial \xi^i} + d \end{aligned}}$$

$$\xi = \begin{pmatrix} x_e \\ p_e \end{pmatrix}$$

$$a = \begin{pmatrix} 1 & 0 \\ 0 & \kappa_e^2 \end{pmatrix} \qquad b = \begin{pmatrix} \kappa_e^2 & 0 \\ 0 & T\bar{T} \end{pmatrix}$$

Note:

- Except for ξ^2 term in L_0, it is the same as ϕ^3 theory in $2N$d dimensions
- b is almost diagonal.

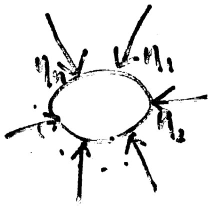
$$T\bar{T} = 1 - \frac{w\bar{w}}{1 + \bar{w} w}$$

$1 + \bar{w} w \to \infty$ as $N \to \infty$ limit
However, this term has nonzero contribution to amplitude

Feynman rule in momentum space

$$\xi \to \eta$$
coordinate momentum

Vertex



$$\Longleftrightarrow \quad e^{-\frac{1}{2}\sum_{i<j}\eta_i \sigma \eta_j}\ \delta^{2dN}(\eta_1 + \cdots + \eta_n)$$

Propagator (Momentum is <u>not</u> conserved)

$$\eta_2 \xrightarrow{\tau} \eta_1 \quad \Longleftrightarrow \quad \langle \eta_2 | e^{-\tau L_0} | \eta_2 \rangle$$

$$= g(\tau, p) \exp\left(-\bar{\eta}_1 F_1 \eta_1 - \bar{\eta}_2 F_2 \bar{\eta}_2 + 2\bar{\eta}_2 G \eta_1 + (\bar{\eta}_1 + \bar{\eta}_2) H\right)$$

$$g(\tau, p) \propto (1 + \bar{w} w)^{\frac{d}{2}} \left(\prod_{e>0} (1 - e^{-2\tau \kappa_e}) \prod_{o>0} (1 - e^{-2\tau \kappa_o}) \right)^{-\frac{d}{2}}$$

$$F(\tau) = \begin{pmatrix} \frac{1}{\kappa_e \tanh(\tau \kappa_e)} & 0 \\ 0 & \frac{1}{4} \bar{R} \kappa_o \frac{1}{\tanh \tau \kappa_o} R \end{pmatrix} \quad G = \begin{pmatrix} \frac{1}{\kappa_e \sinh(\tau \kappa_e)} & 0 \\ 0 & \frac{1}{4} \bar{R} \frac{\kappa_o}{\sinh \tau \kappa_o} R \end{pmatrix}$$

$$H(\tau, p) = \frac{\mathrm{th}(\tau \kappa_e / 2)}{\kappa_e} \frac{w p}{2}$$

External state (<u>not</u> momentum eigenstate)

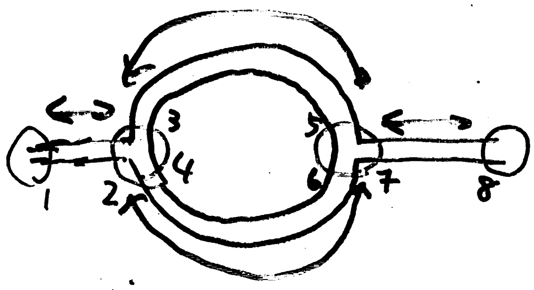
$$\bullet \longrightarrow \eta$$
$A_{N,M,\lambda}$

$$A_{\tilde{N},M,\lambda} = \tilde{N}\, e^{-\frac{1}{4}\bar{\eta} M^{-1} \eta + \frac{i}{2} \bar{\lambda} M^{-1} \eta + i p \bar{x}}$$

$$\tilde{N} = N (4\pi)^{-dN} (\det M)^{-d/2}\, e^{-\frac{1}{4} \bar{\lambda} M^{-1} \lambda}$$

Summary of algorithm

① Decompose string diagram into vertex / propagator / external states



② For each cuts, assign momentum η_i and define gaussian integration as follows

③ Quadratic part of gaussian integration

i-i component

- propagator F
- external state $-\frac{1}{4}M^{-1}$

i-j component

- i-j share same vertex $\pm\frac{1}{2}\sigma$
- " " " propagator $-G$

Linear term

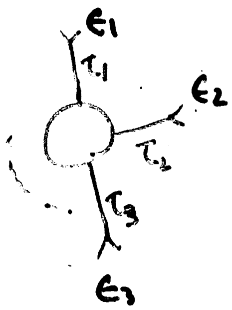
$\begin{cases} \text{propagator} & H \\ \text{external state} & \frac{i}{2}M^{-1}\lambda \end{cases}$

④ Perform gaussian integration

⇒ taking inverse & determinant of above matrix

Reorganization of Gaussian Integration

One may formaly absorb some of the gaussian integration by defining "dressed" vertex & external state

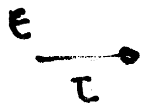


$$= \mathrm{Tr}\left(A_{N_1,M_1,\lambda_1}(\tau_1,\epsilon_1) * \cdots * A_{N_n,M_n,\lambda_n}(\tau_n,\epsilon_n)\right)$$

$$M_i = (4F(\tau_i))^{-1}$$

$$\lambda_i = -\frac{1}{2}F(\tau_i)^{-1}\left(G(\tau_i)\epsilon_i + H(\tau_i, p_i)\right)$$

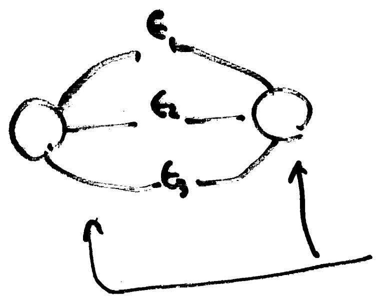
While complicated this quantity is already what we computed



$$= \tilde{A}_{N,M,\lambda}(\tau,\eta)$$

With the help of "dressed" vertex & external state one may formally write Feynmann diagram without propagator.

Example: Two loop vacuum amplitude



dressed 3-string vertex

§6 Exact solution & tachyon condensation

MSFT e.o.m.

$$\boxed{(L_0 - 2\nu) A + A * A = 0}$$

$$L_0 A = \mathcal{L}_0 * A + A * \mathcal{L}_0 + \gamma A$$

$$\mathcal{L}_0 = \sum_{e>0} \left(\tfrac{1}{2} p_e^2 + \tfrac{\kappa_e^2}{2} x_e^2 \right) + i \sum_{o>0} \kappa_o \left(\tfrac{1}{2} x_o y_o + 2 p_o q_o \right) - \frac{d-2}{4} \sum \kappa_n$$

$$\gamma = -\frac{1}{1+\bar{w}w} \left(\sum_{e>0} w_e p_e \right)^2 + 4i (1+\bar{w}w) \left(\sum_{o>0} \kappa_o v_o p_o \right) \left(\sum_{o>0} v_o q_o \right)$$

$\mathcal{L}_0$: propagator for half string

γ : mid-point correction
which explains spectral asymmetry
between $\kappa_e \leftrightarrow \kappa_o$

Symmetric model

$\kappa_e = \kappa_{e-1}$ $\quad T = R = 1 \quad v = w = 0 \Rightarrow \boxed{\gamma = 0}$

open string split exactly into half strings

Study of symmetric model $(T=R=1,\ v=w=0)$

$$\mathcal{L}_0 = \sum_{e,\mu} \kappa_e\, a_e^{\mu\dagger} a_e^{\mu} + \sum_{o>0} \kappa_o (b_o^\dagger c_o + c_o^\dagger b_o)$$

$$a = \frac{1}{\sqrt{2}}\left(\sqrt{\kappa_e}\, x_e + i \frac{p_e}{\sqrt{\kappa_e}}\right), \quad a^\dagger = \frac{1}{\sqrt{2}}\left(\sqrt{\kappa_e}\, x_e - i \frac{p_e}{\sqrt{\kappa_e}}\right)$$

Perturbative vacuum

$$A_0 \sim e^{-\xi M_0 \xi} = e^{-\sum_e \left(\kappa_e x_e^2 + \frac{1}{\kappa_e} p_e^2\right)}$$

$$A_0 * A_0 = A_0 \qquad \text{Vacuum} = \text{"butterfly" projector}$$

$$a_e * A_0 = A_0 * a_e = 0 \qquad A_0 \sim |0\rangle\rangle\langle\langle 0| \quad \left(\begin{array}{c}\text{cf.}\\ \text{Kawano-}\\ \text{Okuyama}\end{array}\right)$$

Excited state (one oscillator example)

$$\phi_{nm} \equiv \frac{1}{\sqrt{n!\,m!}} (a_e)_*^n * A_0 * (a_e^\dagger)_*^m$$

This is orthogonal basis in the following sense

$$\hat{N} = a^\dagger a, \quad \hat{N} * \phi_{nm} = n\,\phi_{nm}, \quad \phi_{nm} * \hat{N} = m\,\phi_{nm}$$

$$\phi_{nm} * \phi_{n'm'} = \delta_{mn'}\,\phi_{nm'}$$

$\downarrow$ multi-oscillator generalization by tensor product

$$\left\{\begin{array}{l} \mathcal{L}_0 * \phi_{IJ} = \lambda_I\, \phi_{IJ} \\ \phi_{IJ} * \mathcal{L}_0 = \lambda_J\, \phi_{IJ} \\ \phi_{IJ} * \phi_{KL} = \delta_{JK}\, \phi_{IL} \end{array}\right. \qquad \left(\begin{array}{l}\phi_{00} \equiv A_0 \\ \quad = \text{butterfly state}\end{array}\right)$$

We expand $A = \sum_{I,J} A_{IJ} \phi_{JI}$

eom $\Leftrightarrow$ $\boxed{(\lambda_I + \lambda_J) A_{IJ} + \sum_K A_{IK} A_{KJ} = 0}$ (*)

Theorem (Reduction to finite n)

> Pick up arbitrary n indices $B = \{I_1, \cdots, I_n\}$
>
> One may consistently put
>
> $$A_{IJ} = 0 \quad \text{if} \quad I \notin B \text{ or } J \notin B$$

Problem reduces to e.o.m for $n \times n$ matrix. $\Rightarrow$ soluble.

We solve (*) for $n = 1$

$$A = -2 \lambda_I \phi_{II} \Leftarrow \text{rank 1 projector}$$

More general solution

$$P = \sum_{I \in B} \phi_{II}, \quad P^2 = P \qquad L_0 * P = P * L_0$$

$$\boxed{A_P = -2 L_0 * P = -2 \sum_{I \in B} \lambda_I \phi_{II}}$$

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Re-expansion around A_P $\quad (\mathcal{L}_0 - \nu \equiv \mathcal{L}_0')$

$$S[A_P + \delta A] = \frac{4}{3}\mathrm{Tr}(\mathcal{L}_0^3 P) \quad \leftarrow \text{tension of (un)stable D-brane}$$

$$+ \mathrm{Tr}(\mathcal{L}_0 - A_P) * \delta A * \delta A \Leftarrow \text{modified kinetic term}$$

$$+ \frac{1}{3}\mathrm{Tr}(\delta A)^3$$

$$\text{Tension} = \frac{4}{3}\sum_{I\in B}(\lambda_I - \nu)^3$$

$$\mathcal{L}_{AP} \equiv \mathcal{L}_0 - A_P = \sum_I (\lambda_I - \nu)(1 - 2\theta_B(I))\,\phi_{II}$$

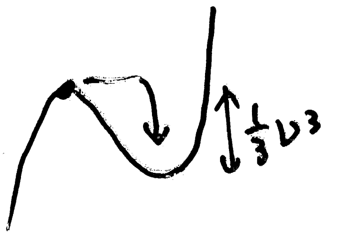
$$\theta_B(I) = \begin{cases} 1 & I \in B \\ 0 & I \notin B \end{cases}$$

m^2 of half string changes sign when $I \in B$

Tachyon $\longleftrightarrow$ massive particle

Scenario

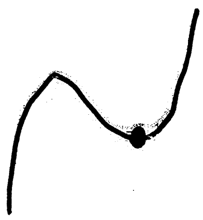
$\nu > 0$ $\quad \lambda_I - \nu < 0$ for $I = 0$
$(\lambda_0 \equiv 0)$



$S[A_P] = -\frac{4}{3}\nu^3$
$\uparrow$
Butterfly state

Tachyon $\rightarrow$ massive particle

$\nu \leq 0$



all solution corresponds to unstable D-brane

$A = 0$ is tachyon vacuum

§7 To do list

- Quest for exact vacuum
  - inclusion of γ-term
  - Nature of tachyon vacuum
  - Examination of VSFT ansatz

Might be difficult analytically.
Intermediate step...

  - Numerical study
  - Taylor expansion from $\gamma = 0$

- Derivation of Venziano amplitude
  including algebraic manipulation of $T, R, \ldots$ algebra

- Treatment of $\begin{pmatrix} \text{BRST} \\ \text{conformal} \end{pmatrix}$ symmetry
  - How to fix K_e, K_o ?
  - Negative energy state at finite N ?

- SUSY extension
  Matrix model analogy
  Nontrivial Background
  ⋮

BKMn ($n \geq 3$) to appear???

Concluding Remark

- Toward M-theory
  - Membrane (Hayakawa)
  - E_{10} symmetry (Mizoguchi)
- Nonperturbative formulation
  - Matrix model (Kuroki)
  - String Field Theory
  - Noncommutative geometry (Okawa)
- Holography
  - AdS/CFT (Kobayashi)
  - pp wave (Yoneya Imamura)
  - DV (Itoyama)
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  - Black hole.
  - Big crunch, big bang
  - Rolling tachyon (Hashimoto)

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お疲れ様でした、

皆様 良いお年を