Flavor structure in string models

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1. Introduction
2. Yukawa in heterotic orbifold models
3. Yukawa in intersecting D-brane models
4. Discrete flavor symmetry
5. Summary

based on Ko, T.K. and Park, hep-ph/0406041, hep-ph/0502nnn
Kitazawa, T.K., Maru and Okada, hep-th/0406115
T.K., Raby and Zhang, hep-ph/0409098, work in progress
Higaki, Kitazawa, T.K. and Takahashi, work in progress

1. Introduction

The origin of fermion masses and mixing angles is one of important issues in particle physics.

Fermion masses ← Yukawa couplings between fermions and Higgs fields e.g. in the Standard Model

O(1) of Yukawa couplings are in a sense natural.

From this viewpoint, how to derive suppressed Yukawa couplings is a key-point in understanding fermion masses.

Quark masses and mixing angles

$$M_t = 174 \quad GeV, \qquad M_b = 4.3 \quad GeV$$
$$M_c = 1.2 \quad GeV, \qquad M_s = 117 \quad MeV$$
$$M_u = 3 \quad MeV, \qquad M_d = 6.8 \quad MeV$$

$$V_{us} = 0.22, \qquad V_{cb} = 0.04, \qquad V_{ub} = 0.004$$

RG effects are not drastic in usual cases except PR-fixed points or superconformal fixed points.

Lepton masses and mixing angles

$$M_e = 0.5 \quad MeV, \qquad M_\mu = 106 \quad MeV$$

$$M_\tau = 1.8 \quad GeV,$$

$$\Delta M_{21}^2 = 8 \times 10^{-5} \quad eV^2, \qquad \Delta M_{31}^2 = 2 \times 10^{-3} \quad eV^2$$

$$\sin^2\theta_{12} = 0.3, \qquad \sin^2\theta_{23} = 0.5, \qquad \sin^2\theta_{13} = 0.00,$$

RG effects are not drastic in usual cases except PR-fixed points or superconformal fixed points.

String phenomenology

String theory promising candidate for unified theory
including gravity

It is important to study flavor structure and Yukawa structure from string theories.

Superstring theory → 4D space-time
+ 6D compact space

6D compact space → Flavor (Yukawa) structure

String models (N=1 or 0 SUSY)

Hetero. on CY manifolds, orbifolds
fermionic const., Gepner…

Intersecting D-brane models

(Magnetized D-brane models)

Flavor structure from 6D compact space

6D compact space consistent with string theory

How to count the family number is well-known.

Flavor (Yukawa) structure

→ selection rule for allowed Yukawa couplings

calculations of their magnitude

flavor symmetry

→ Classify possible patterns of Yukawa matrices for quarks and leptons

Heterotic orbifold models and Intersecting D-brane models have similarlities.

We study them parallel (and systematically).

Localized modes

Heterotic orbifold models and

Intersecting (magnetized) D-brane models

are interesting to derive suppressed Yukawa couplings, because they have localized models.

Heterotic orbifold models

gauge bosons ← 10D (bulk) modes

(untwisted) matter fields ← 10D (bulk) modes

twisted matter fields ← 4D, 6D modes

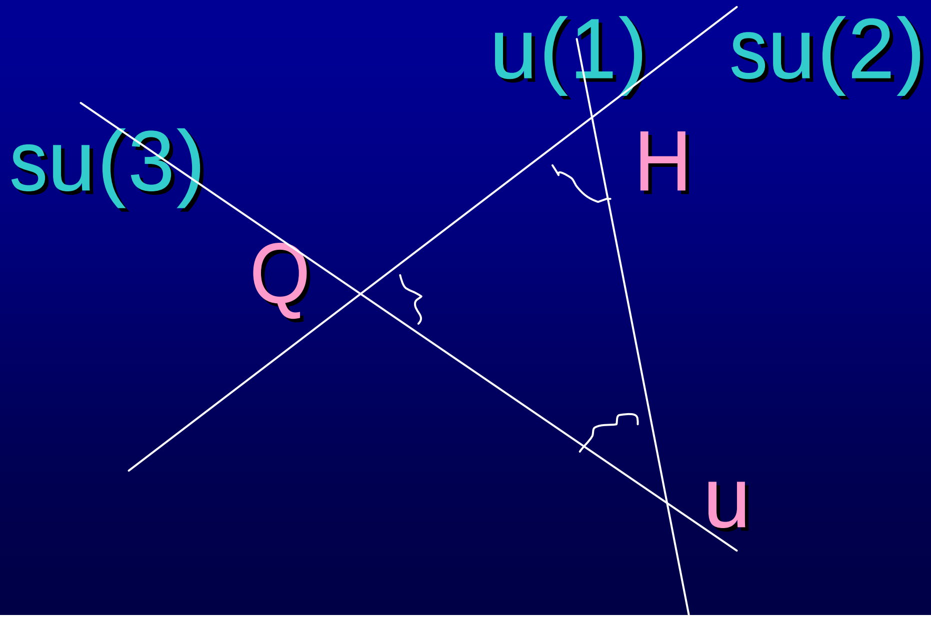
on fixed points, torus

Intersecting D-brane models
(D6 branes on orbifold)

gauge bosons: (p+1) D modes on D-brane

(bi-fundamental)

matter fields: modes localized at intersecting points

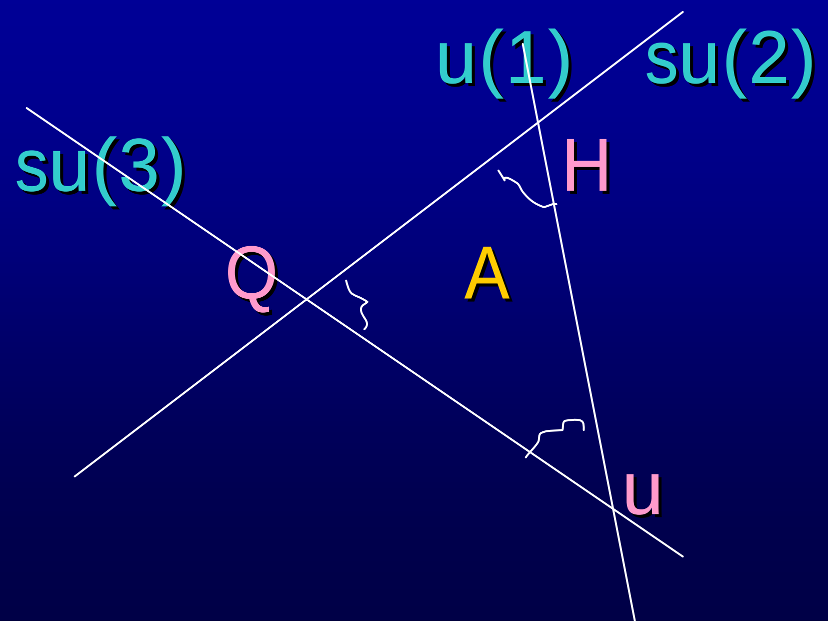


Non-universal (suppressed) and/or moduli-dependent Yukawa couplings

e.g., intersecting D-brane models

hetero. models on orbifolds

Y = exp (-A)   A: area



2. Yukawa in hetero. orbifolds

Dixon, et. al., '85

The number of 6D orbifolds is finite,

$Z_3, Z_4, Z_6, \ldots\ldots$

(crystallographic) space group

For an orbifold all of fixed points, where twisted string is localized, are known,

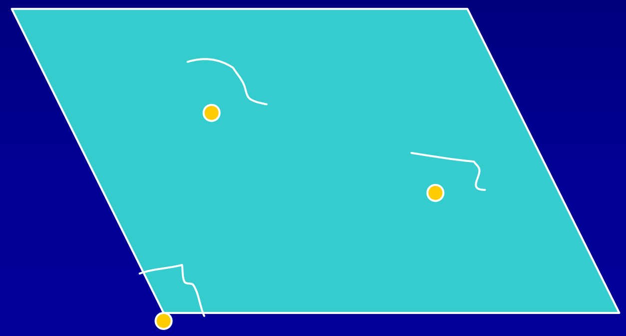
and its number is finite.

We know selection rules for allowed Yukawa couplings, and their magnitudes.

In principle, systematic study is possible.

Heterotic orbifold models

Orbifold



Fixed points on 2D Z_3 orbifold

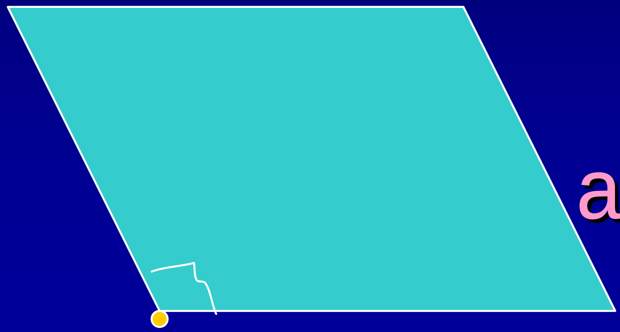
(0,0), (2/3,1/3), (1/3,2/3) in su(3) root basis

Twisted strings are associated with these fixed points.

The flavor structure and the selection rule are not so model-dependent compared with intersecting brane models, and these are determined by geometrical aspect when we fix orbifold.

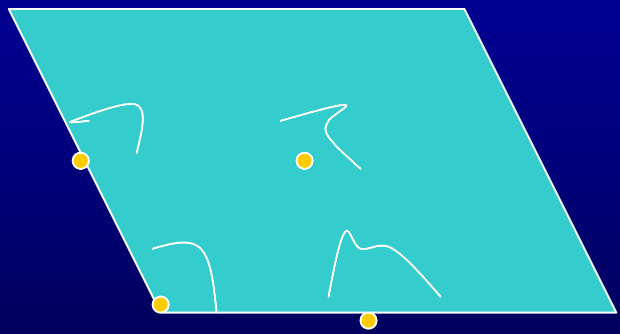
2D Z6 orbifold

First twisted states T1



a single fixed point (0,0)

Second twisted states T2 third ones T3



(0,0), (2/3,1/3), (1/3,2/3) (0,0), (1/2,0),(1/2,1/2)(0,1/2)

6D orbifold

6D Z3 orbifold = a product of 3 (2D Z3)

27 fixed points

6D Z6-I orbifold =

a product of 2 (2D Z6) and (2D Z3)

T1 3 twisted states

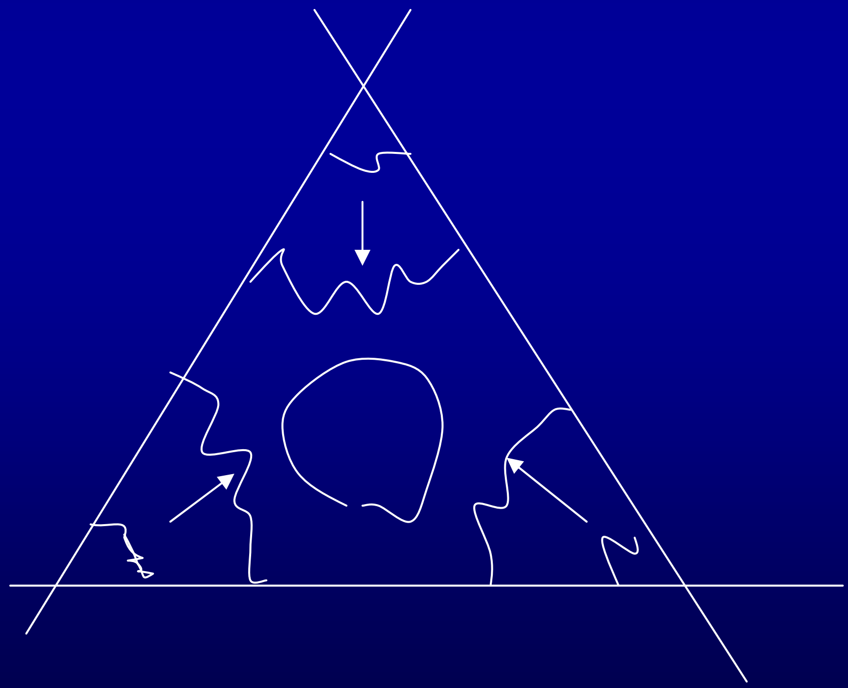
T2 27 twisted states

T3 16 twisted states

Coupling selection rule

In general, stringy selection rules are very tight, and it is not easy to derive off-diagonal couplings.

Yukawa coupling is allowed when three twisted strings are combined into a closed string.



Space group selection rule

$$X(\sigma=\pi)=\Theta X(\sigma=0)+e$$

$$(\Theta,e)(\Theta',e')(\Theta'',e'')=(1,0)$$

Conjugacy class

$$X+\Lambda=\Theta(X+\Lambda)+e$$

$$(\Theta,e)=(\Theta,e+(1-\Theta)\Lambda)$$

In some case, only diagonal couplings are allowed, and in other case off-diagonal couplings are also allowed.

Space group selection rule

2D Z_3 Only T1 T1 T1 couplings are allowed.

f_k k=0,1,2

$i + j + k = 0 \mod 3$ diagonal

We can not get non-vanishing mixing angles.

This corresponds to Z3 symmetry, and its charge is equal to k.

2D Z6 orbifold

Only T1 T2 T3 and T2 T2 T2 couplings are allowed.

Off-diagonal couplings are allowed for T1T2T3.

T2 T2 T2 couplings are diagonal.

For example, T1T1T1 couplings are not allowed.

Yukawa couplings on orbifold

CFT calculations → Dixon, et al '87, Hamidi, Vafa, '87, Burwick, et al '91, Casas, et al '93……
T.K., Lebedev '03

$\hat{T}_1\hat{T}_2\hat{T}_3$ *couplings*

$$Y = \exp[-\sqrt{3}/(4\pi)(f_2 - f_3)^T M (f_2 - f_3)]$$

$$M = \begin{pmatrix} (R_1)^2 & -3(R_1)^2/2 & 0 & 0 \\ -3(R_1)^2/2 & 3(R_1)^2 & 0 & 0 \\ 0 & 0 & (R_2)^2 & -3(R_2)^2/2 \\ 0 & 0 & -3(R_2)^2/2 & 3(R_2)^2 \end{pmatrix}$$

$\hat{T}_2\hat{T}_2\hat{T}_2$ *couplings*

$$Y = \exp[-\sqrt{3}/(16\pi)(f_2 - f_3)^T M (f_2 - f_3)]$$

Systematical study on Z6-I: Quark sector

Ko, T.K., Park '04

We study systematically all of possible assignments of 2^{nd} and 3^{rd} families to fixed points,

assume one pair of up and down Higgs fields,

examine allowed entries by the selection rule,

try to fit the mass ratios m_c/m_t, m_s/m_b and

mixing angle V_{cb} by varying R_1 and R_2.

$$[m_c / m_t]_{\exp} = 0.0038, \qquad [m_s / m_b]_{\exp} = 0.025$$

$$[V_{cb}]_{\exp} = 0.041$$

Assignment 1

$Q_2, Q_3,$ u_2, u_3 d_2, d_3 H_u, H_d

$\hat{T}_2^{(2)}, \hat{T}_2^{(4)}$ $\hat{T}_3^{(3)}, \hat{T}_3^{(2)}$ $\hat{T}_3^{(1)}, \hat{T}_3^{(3)}$ $\hat{T}_1, \hat{T}_1$

$T_1 = 27.8,$ $T_2 = 107$

$$Y_u = \begin{pmatrix} 0.0416 & 0.718 \\ 0.0557 & 0.848 \end{pmatrix} \qquad Y_d = \begin{pmatrix} 0.0313 & 0.0416 \\ 0.0370 & 0.0557 \end{pmatrix}$$

$$m_c / m_t = 0.0038, \qquad m_s / m_b = 0.029, \qquad V_{cb} = 0.041$$

There are several examples leading to similar results.

Assignment 2

$Q_2, Q_{3,}$ u_2, u_3 d_2, d_3 H_u, H_d

$\hat{T}_3^{(2)}, \hat{T}_3^{(4)}$ $\hat{T}_2^{(3)}, \hat{T}_2^{(2)}$ $\hat{T}_2^{(1)}, \hat{T}_2^{(3)}$ $\hat{T}_1, \hat{T}_1$

$T_1 = 24.0,$ $T_2 = 150$

$$Y_u = \begin{pmatrix} 0.0281 & 0.439 \\ 0.0371 & 0.665 \end{pmatrix} \qquad Y_d = \begin{pmatrix} 0.0199 & 0.0281 \\ 0.0302 & 0.0371 \end{pmatrix}$$

$m_c / m_t = 0.0038$, $m_s / m_b = 0.032$, $V_{cb} = 0.041$

There are several examples leading to similar results.

Assignment 5

$Q_2, Q_3,$ $\quad u_2, u_3$ $\quad d_2, d_3$ $\quad H_u, H_d$

$\hat{T}_2^{(2)}, \hat{T}_2^{(4)}$ $\quad \hat{T}_2^{(2)}, \hat{T}_2^{(4)}$ $\quad \hat{T}_2^{(3)}, \hat{T}_2^{(2)}$ $\quad \hat{T}_2^{(4)}, \hat{T}_2^{(4)}$

$T_1 = 180,$ $\quad T_2 = 180$

$$Y_u = \begin{pmatrix} 0 & 0.0309 \\ 0.0309 & 0.500 \end{pmatrix} \qquad Y_d = \begin{pmatrix} 0.00132 & 0 \\ 0.0214 & 0.0309 \end{pmatrix}$$

$$m_c / m_t = 0.0038, \qquad m_s / m_b = 0.029, \qquad V_{cb} = 0.041$$

Results

We have found examples leading to a realistic mixing angle as well as mass ratios between the 2nd and 3rd families of quarks.

Our results is the first examples to show the possibilities for leading to realistic mixing angles by use of stringy renomalizable couplings with one pair of the up and down Higgs fields.

Results for three families of quarks are not good.

Systematical study on Z6-I: Lepton sector

Ko, T.K., Park '05

We study systematically all of possible assignments of three families to fixed points,

assume one pair of up and down Higgs fields,

examine allowed entries by the selection rule,

try to fit the lepton mass ratios and

mixing angles by varying R_1 and R_2.

$$[m_e / m_\tau]_{\exp} = 0.000288, \qquad [m_\mu / m_\tau]_{\exp} = 0.0595$$

$$[\Delta m_{31}^2 / \Delta m_{21}^2]_{\exp} = 27, \qquad [\sin^2\theta_{12}]_{\exp} = 0.30$$

$$[\sin^2\theta_{23}]_{\exp} = 0.50, \qquad [\sin^2\theta_{13}]_{\exp} = 0.000$$

Dirac neutrino mass: Assignment 4

$L_1, L_2, L_{3,}$ N_1, N_2, N_3 e_1, e_2, e_3 H_u, H_d

$\hat{T}_2^{(2)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,-1)}$ $\hat{T}_2^{(2)}, \hat{T}_2^{(4,1)}, \hat{T}_2^{(4,-1)}$ $\hat{T}_3^{(1)}, \hat{T}_3^{(2)}, \hat{T}_3^{(4,1)}$ $\hat{T}_2^{(2)}, \hat{T}_1$

$T_1 = 26, \quad T_2 = 21$

$m_e / m_\tau = 0.0003, \quad m_\mu / m_\tau = 0.06, \quad \Delta m_{31}^2 / \Delta m_{21}^2 = 14,$

$\sin^2\theta_{12} = 0.38, \quad \sin^2\theta_{23} = 0.70, \quad \sin^2\theta_{13} = 0.000,$

There are several examples leading to similar results、 but smaller ratios of neutrino mass difference.

Seesaw scenario: Assignment 4

We assume the right-handed Majorana neutrino mass to be proportional to identity matrix for simplicity.

$L_1, L_2, L_{3,}$ N_1, N_2, N_3 e_1, e_2, e_3 H_u, H_d

$\hat{T}_2^{(2)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,1)}$ $\hat{T}_2^{(2)}, \hat{T}_2^{(3)}, \hat{T}_2^{(4,1)}$ $\hat{T}_3^{(1)}, \hat{T}_3^{(2)}, \hat{T}_3^{(4,1)}$ $\hat{T}_2^{(2)}, \hat{T}_1$

$T_1 = 23, \quad T_2 = 26$

$m_e / m_\tau = 0.0003, \quad m_\mu / m_\tau = 0.06, \quad \Delta m_{31}^2 / \Delta m_{21}^2 = 29,$

$\sin^2\theta_{12} = 0.32, \quad \sin^2\theta_{23} = 0.48, \quad \sin^2\theta_{13} = 0.000,$

There are several examples leading to similar results.

Results

We have found examples leading to a realistic values of charged lepton mass ratios, neutrino mass difference ratio and mixing angles for three families.

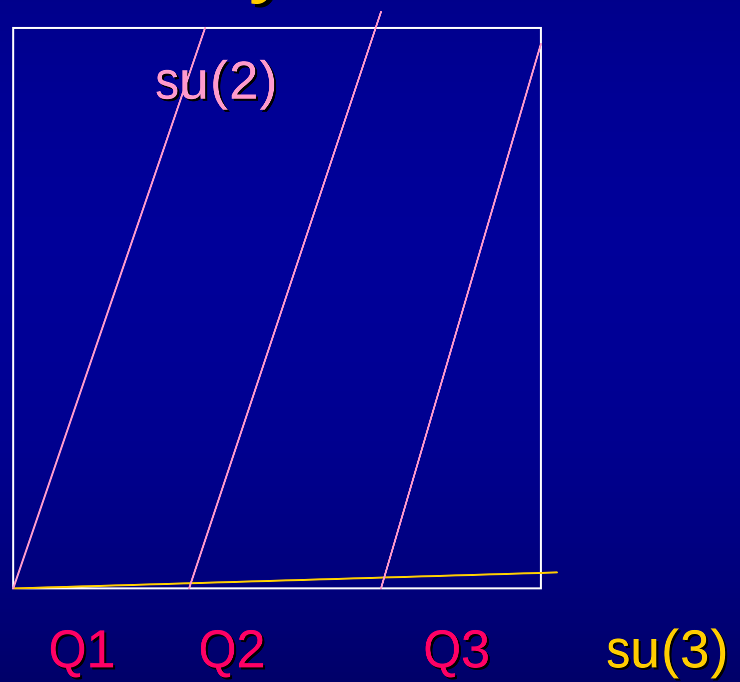
It is quite non-trivial to fit six observables only by two parameters.

We also have some reasoning of strongly suppressing neutrino Yukawa in Dirac neutrino mass scenario and deriving right-handed majorana neutrino mass proportional identity in the seesaw scenario.

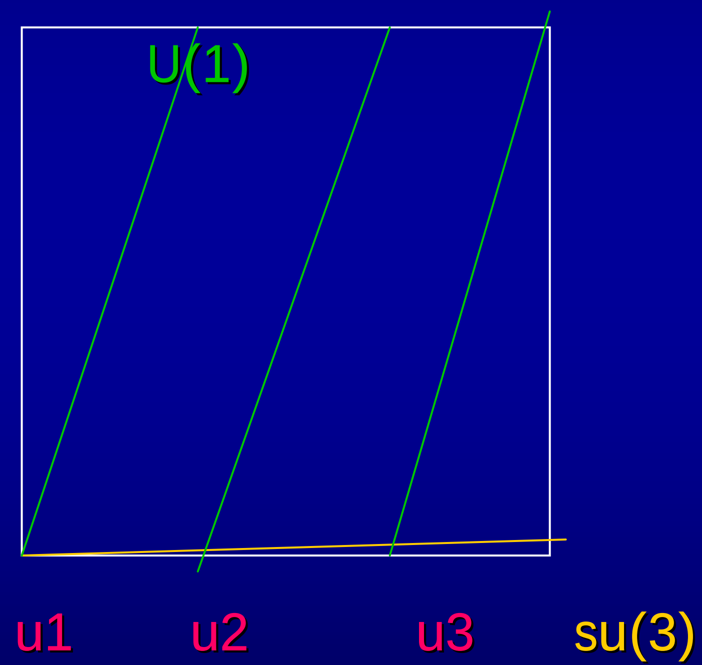
3. Yukawa in intersecting D6 models

Origin of flavor

Family number = intersection number



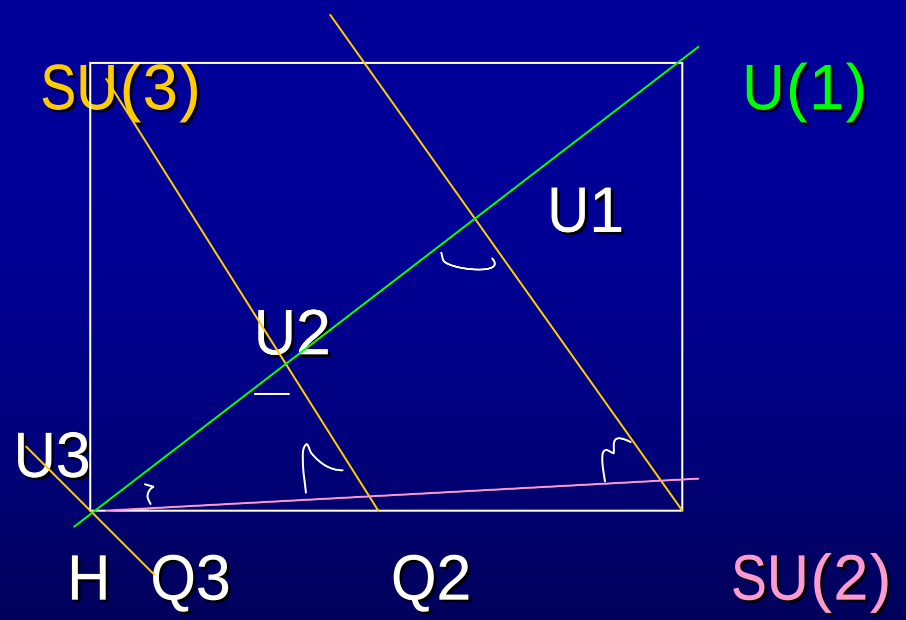
1st plane



2nd plane

Yukawa in intersecting D-brane models

Flavor number = intersecting number



Flavor structure

Flavor number = intersecting number

Model-dependent calculation

-> non-realistic Yukawa matrices,

We need to formulate model-independent analysis on selection rules for allowed Yukawa couplings like heterotic orbifold models.

We would like to study somehow systematically

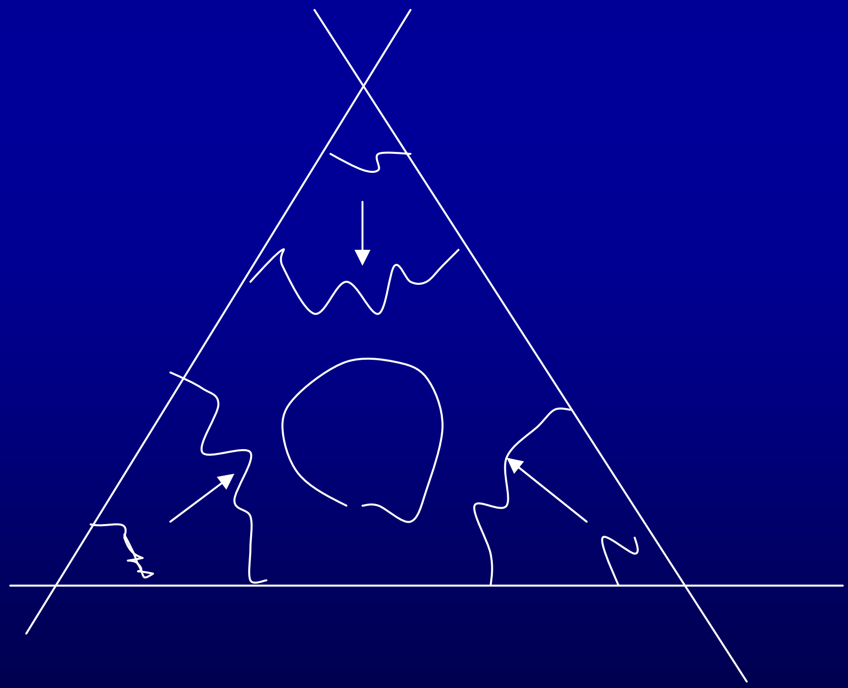
flavor structure, which can be realized in intersecting D-brane configuration.

(work in progress with Higaki, Kitazawa, Takahashi)

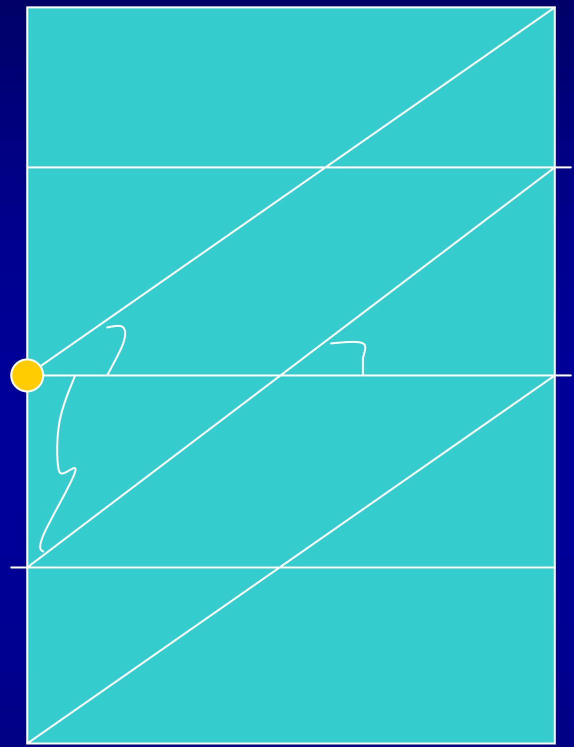
Coupling selection rule

Yukawa coupling is allowed when three twisted strings are combined into a closed string.

We have to consider the equivalence on torus.



Open string at intersecting D-brane



How can we describe open strings at different intersecting points and open strings equivalent up to torus ?

Intersecting points

Two D-brane Da, Db (winding numbers)

$$W_a = (n_a, m_a), \qquad W_b = (n_b, m_b),$$

Intersecting number Iab

Intersecting points

$$(k/I_{ab})W_a, \qquad (\ell/I_{ab})W_b, \qquad k,\ell = 0,1,\ldots,I_{ab}-1$$

$$(k/I_{ab})W_a = (\ell/I_{ab})W_b + V_{ab}$$

Equivalent intersecting point sets with Vab on lattice

Shift vectors represent intersecting points up to

$$V_{ab} = V_{ab} + k_a W_a + k_b W_b$$

$$\Lambda_{ab} = \{W_a, W_b\}, \qquad \Lambda = \{(1,0),(0,1)\}$$

Example

Two D-brane Da, Db (winding numbers)

$$W_a = (1,0), \qquad W_b = (1,3),$$

Intersecting number Iab=3

Intersecting points

$$(k/3)W_a, \qquad k = 0,1,2$$

Shift vectors represent three intersecting points.

$$V_{ab} = (0,0),(0,1),(0,2), \qquad \Lambda_{ab} = \{(1,0),(0,3)\},$$

Open strings and selection rule

End points of open string

$$X(\sigma=\pi)=X(\sigma=0)+V_{ab}$$

Three sets of intersecting D-branes

$$W_a=(n_a,m_a), \qquad W_b=(n_b,m_b), \qquad W_c=(n_c,m_c),$$

Condition for Yukawa couplings among three open strings between Da-Db, Db-Dc and Dc-Da branes

$$V_{ab}+V_{bc}+V_{ca}=0$$

$$\text{mod} \qquad \Lambda_{ab},\Lambda_{bc},\Lambda_{ca}$$

only diagonal couplings or off-diagonal couplings ?

Flavor structure

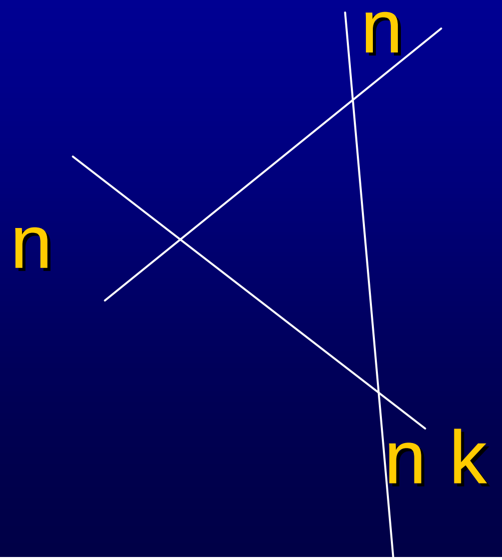
Which types of flavor structure can be realized from intersection D-brane configurations ?

← theory of numbers, elementary vector analysis ?

infinite number of varieties ?

Left-right (n,n) symmetric generation on torus

→ the number of Higgs scalars = n k



for k=1

$$\Lambda_{ab} = \Lambda_{bc} = \Lambda_{ca}$$

diagonal couplings

Flavor structure

e.g. 3 families → 3 Higgs

Left-handed Q $\quad V_{ab} = (0,0), (0,1), (0,2)$

Right-handed Q $\quad V_{bc} = (0,0), (0,1), (0,2)$

Higgs $\quad V_{ca} = (0,0), (0,1), (0,2)$

$$V_{ab} + V_{bc} + V_{ca} = (0,0) \qquad \mathrm{mod} \qquad (0,3)$$

Selection rule for Yukawa coupling ← Z3 symmetry

diagonal couplings like Z3 heterotic orbifold models

Yukawa matrix

$$Y = \begin{pmatrix} H_0 & \varepsilon H_2 & \varepsilon H_1 \\ \varepsilon H_2 & H_1 & \varepsilon H_0 \\ \varepsilon H_1 & \varepsilon H_0 & H_2 \end{pmatrix}$$

Quark masses

$$(m_u, m_c, m_t) \propto (v_0^u, v_1^u, v_2^u)$$

$$(m_d, m_s, m_b) \propto (v_0^d, v_1^d, v_2^d)$$

Mixing angles

$$V_{12} : V_{13} : V_{23} = m_t / m_c : m_s / m_b : m_d / m_b$$

→ Not realistic

Result

We have formulated the selection rule for allowed Yukawa couplings.

Left-right symmetric 3 families

→ more than 3 Higgs → unrealistic mixing angles

We need another D-brane configuration.

We are considering left-right asymmetric D-brane cofigurations.

4-2. Composite approach

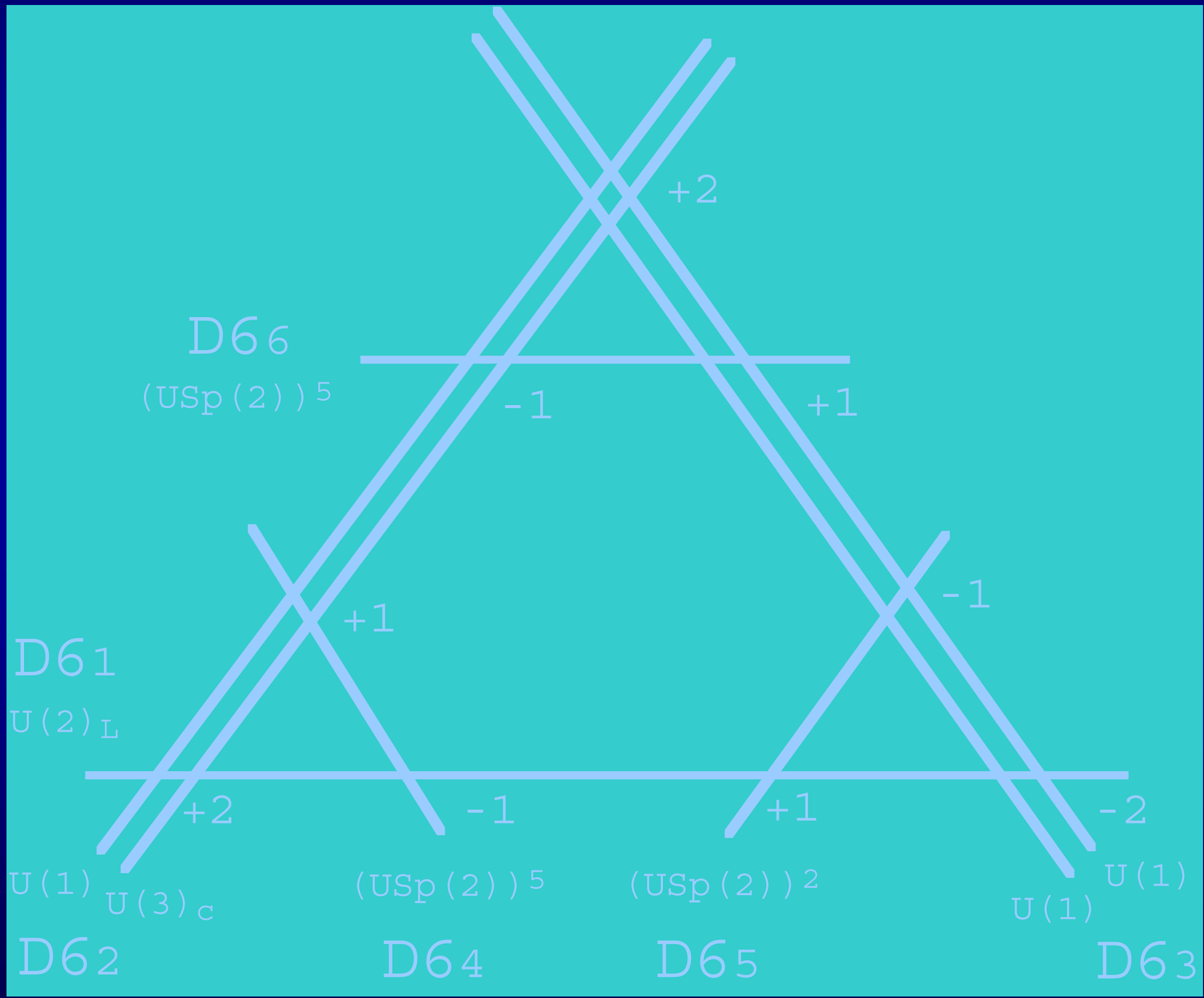
3-point couplings → unrealistic Yukawa
in intersecting D-brane (no mixing)

We need another approach.

How about composite approach ?

Composite model

Kitazawa, '04



(Bifundamental) matter

Intersecting number

SU(3)	SU(2)	-2	anti-generation
SU(3)	$USp(2)_a$	1	C_a
SU(2)	$USp(2)_a$	1	D_a

$C_a D_a \to Q_a$ (a=1,…,6)

Yukawa matrices

Kitazawa,T.K.,Maru,Okada '04

$$Y_d = \begin{pmatrix} \varepsilon_3 & \varepsilon_1\varepsilon_3 \\ 0 & \varepsilon_1 \end{pmatrix} \qquad Y_u = \begin{pmatrix} \varepsilon_1\varepsilon_3 & \varepsilon_1^2\varepsilon_3 \\ 0 & 1 \end{pmatrix}$$

$$m_c / m_t = \varepsilon_1\varepsilon_3, \qquad m_s / m_b = \varepsilon_3 / \varepsilon_1, \qquad V_{cb} = \varepsilon_3$$

$$\varepsilon_1 = 0.5, \qquad \varepsilon_3 = 0.01 \rightarrow \textit{realistic}$$

4. Discrete flavor symmetry

Effective Yukawa

← higher dim. op. through symmetry breaking → small Yukawa

e.g. Froggatt-Nielsen

Symmetries are important to control higher dim. Op.

e.g. U(2), S3, D4, A4, U(1), ZN,………

(Symmetrical approach)

What is the origin of these symmetries ?

Discrete Flavor Symmetry

What are their origins of discrete flavor symmetry like S3, D4, A4 …… ?

They are symmetries of geometric solids.

So, they may be originated from geometry of extra dimensional space.

Actually, we have found explicit string models, whose three families = singlet + doublet under D4.

T.K., Raby, Zhang, '04

This is the first explicit models leading to D4 flavor structure in string models.

4-1. D4 flavor

Our model (T.K., Raby, Zhang) is from Z6-II orbifold.

Z6=Z2*Z3

The essence that our model leads to D4 flavor structure can be understood by the simplified extra dim. space, S1/Z2.

There are two fixed points, on which two types of states exist.

In string models, these states are degenerate in massless spectra. These correspond to D4 doublets and bulk modes correspond to singlets.

D4 Flavor Symmetry

Stringy symmetries require that

Lagrangian has the permutation symmetry between 1 and 2, and each coupling is controlled by Z2 symmetry.

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

D4 elements

$$\pm 1, \qquad \pm \sigma_1, \qquad \pm i\sigma_2, \qquad \pm \sigma_3$$

Geometry of compact space

→ origin of finite flavor symmetry

Z6 example

Gauge shift 6V=(22200000)(11000000)

Wilson lines 3W=(-11000000)(00200000)

Wilson line 2W=(10000111)(00000000)

→

Gauge group SU(4) SU(2) SU(2) (hidden group)

Matter 3 [(4,2,1) + ($\bar{4}$,1,2)] + (1,2,2) + exotics

Pati-Salam model with 3 families + exotics

3 families = (D4 doublets + D4 singlets)

Electroweak higgs = D4 singlet

4-2 Yukawa matrices

As renormalizable couplings, the Yukawa couplings between one family (D_4 singlets) and the Higgs fields (D_4 singlets) are allowed.

The other diagonal entries are allowed, but such pattern is not realistic. Thus, we assume such entries are forbidden by extra symmetries.

At the renormalizable coupling level,

$$y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & g \end{pmatrix}$$

In this model, there are singlets to generate the other entries through higher dim. operators with their VEVs.

Effective Yukawa from higher dimensional operators

Singlets can develop their VEVs S

These VEVS lead to effective Yukawa matrices

$$y = \begin{pmatrix} S^a & * & * \\ * & * & * \\ * & * & g \end{pmatrix}$$

We obtain non-trivial Yukawa matrices.

In general, the number of free parameters is large.

No prediction from only D4.

If we assume certain conditions on S,……

Summary

We have studied flavor structure in string models.

Heterotic orbifold models have possibilities for realizing realistic Yukawa matrices for quarks and leptons.

Other orbifolds ?

We have formulated selection rule of Yukawa couplings in intersecting D-brane models.

We have just started the classification of flavor structure derived from D-brane configurations.

We have just started how to derive non-abelian discrete flavor symmetries from string models.

Future study

How to stabilize moduli VEVs at proper values is an important issue to study.

Flavor structure → (realistic) Yukawa matrices

Such flavor structure affects on SUSY breaking terms, e.g. scalar masses and A-matrices.

Each model would have certain pattern of SUSY breaking terms.

It is important to study such prediction on SUSY breaking terms.