Holography in the BMN limit of AdS/CFT Correspondence

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◇ BMN limit from the viewpoint of correlation functions

How to reconcile the BMN conjecture with holographic principle?
– One of open problems in the pp-wave limit –

In particular, we show
how the GKP-W relation is realized in the BMN limit,
and
give a concrete holographic relation for 3-point OPE coefficients.

Based on

- S. Dobashi and T. Yoneya, hep-th/0406225, hep-th/0409058
  (also previous work, S. Dobashi, H. Shimada and T. Yoneya hep-th/0209251)

Contents :

- Motivation : puzzles, and resolution
- Holographic relation for 3-point correlators
- Holographic string-field theory
- Explicit checks in the large-μ limit
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- Concluding remarks
  - Nonconformal Dp-brane backgrounds
  - M-theory holography $\mathrm{AdS}_4 \times S^7$

Motivation

- AdS holography :

Typically, $\mathrm{AdS}_5 \times S^5 \Leftrightarrow \mathcal{N} = 4$ SYM_4

◇ provides a concrete example of 'holographic' correspondence between gravity (string theory) [← bulk] and gauge theory [← boundary]

◇ opens an entirely new perspective both for gauge theories and for string theories

GKP-W relation (conjecture) : Basic holographic relation

$$\text{bulk } (\mathrm{AdS}_5 \text{ or } \mathrm{EAdS}_5) \quad \leftrightarrow \quad \text{boundary } (\mathrm{M}_{3,1} \text{ or } \mathrm{R}_4)\,(z \to 0)$$

$$Z[\Phi_0]_{\text{string}\cdot\text{gravity}} = \langle \exp\Big(\int d^4x \sum_i \Phi_0^i(x)\mathcal{O}_i(x)\Big)\rangle_{\text{SYM}},$$

$$\lim_{z\to 0} \Phi^i(z,x) \to z^{4-\Delta_i}\Phi_0^i(x)$$

Has been checked for protected supergravity modes (chiral primary fields) for 3-point functions

- However, whether this relation is also valid for nonprotected stringy degrees of freedom has been an open question.

BMN proposal as an important first step:

◇ proposed a set of local gauge-invarinat operators on the gauge theory side (CFT) corresponding to stringy massive states of bulk (closed) string theory in the so-called pp-wave limit.

◇ pp-wave limit $\sim$ approximating the AdS geometry by a neighborhood around a null geodesic which describes a ray of center-of-mass motion of strings with large angular momentum $(J \gg 1)$ along a large cirlce of S^5

ϕ_i $(i = 1, \ldots, 6)$: scalar fields ($\sim$ collective coordinates of D3-branes) of SYM$_4$

– ground state $|p^+\rangle$:

$$\mathcal{O} \sim \mathrm{Tr}[Z^J], \quad Z = \phi_5 + i\phi_6$$

– stringy excitation modes ($n \sim$ world-sheet momentum):

$$a_n^{i+4,\dagger} \leftrightarrow Z^\ell \phi_i \mathrm{e}^{2\pi i n\ell/J} Z^{J-\ell}, \quad a_n^{i,\dagger} \leftrightarrow Z^\ell (D_i Z) \mathrm{e}^{2\pi i n\ell/J} Z^{J-\ell} \quad (i = 1, 2, 3, 4)$$

etc

The scalar fields ϕ_i's and derivatives D_i are called *"impurities"*.

BMN operators $\sim$ *'near' BPS* states.

8 transverse string excitations in the bulk

$\Updownarrow$

4 (= $SO(4)$ R-charge directions) + 4 (= $SO(4)$ *base space directions* of SYM)

$J \to \infty$, keeping J/R^2 fixed

$\Downarrow$

Anomalous conformal dimensions Δ are given by

$$\Delta - J = \sum_{\{i\}} N_i \sqrt{1 + \frac{R^4 n_i^2}{J^2}} \Leftrightarrow P^- R, \quad P^+ \sim J/R$$

and are proposed to be identified with
energy spectrum for strings in the light-cone gauge

$\left(R = (g_{YM}^2 N)^{1/4} \sqrt{\alpha'} \sim \text{scale parameter of AdS geometry} \right)$

Puzzles related to holography

- The null geodesics dominating the pp-wave limit do not reach the conformal boundary of AdS space

Impossible to apply the GKP-W relation to the BMN operators !?

- $\tau \leftrightarrow \tau_r$, $x_4 = e^{\tau_r}\hat{x}_4$, of radial quantization on the boundary ?
  - 4 base-space directions of SYM $\in$ 8 transverse directions of bulk (irrespectively of the identification of the global time and the target time on the boundary):

    light-cone time $x^+ = \tau$ mixed with transverse directions !?
  - The null geodesic requires *Minkowski* metric, while the boundary theory must be assumed to be *Euclidean !?*

These puzzles must be resolved:

◇ The spectrum of conformal dimensions alone are not sufficient for defining CFT.

What about the correlation functions, in particular, OPE coefficients ?

Our approach: *Study the large-J limit of GKP-W relation directly*!

we propose

- a simple resolution of all these puzzles related to the identification of the boundary in the pp-wave limit

- a general holographic relation such that

OPE coefficeints C_{ijk} of BMN operators on the CFT side

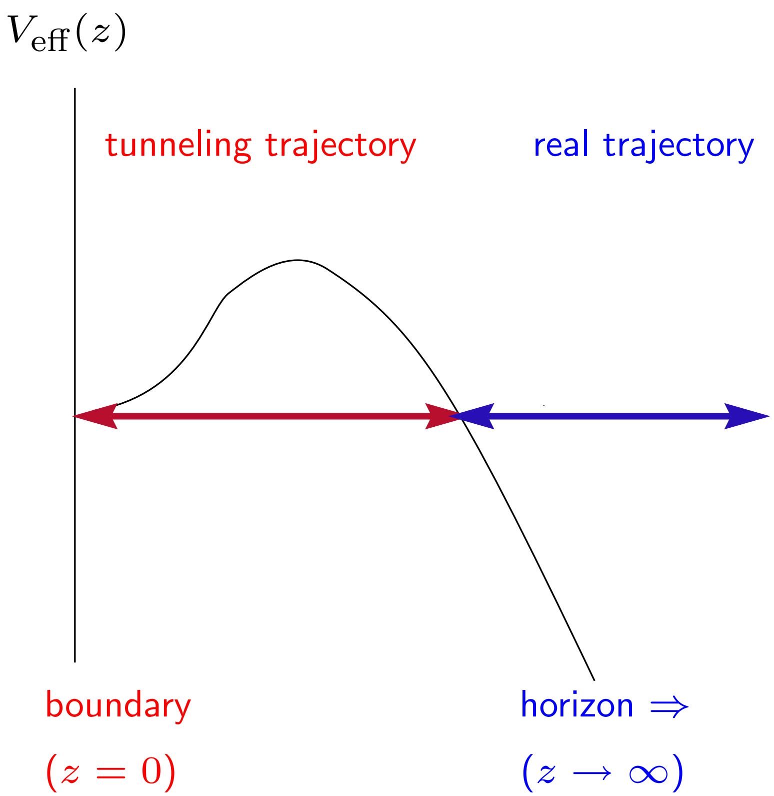
$$\Updownarrow$$

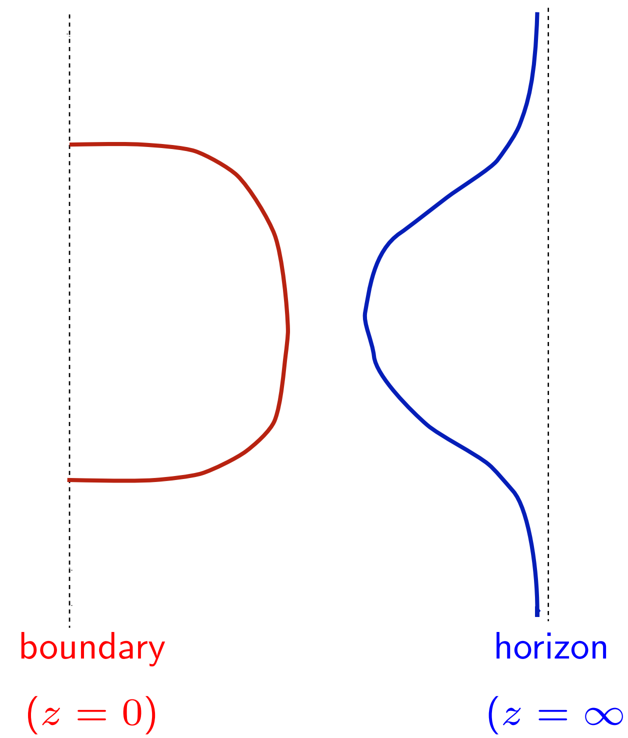
interaction vertex λ_{ijk} of string theory in the bulk pp-wave geometry

Our results are based on a definite spacetime tunneling picture

Dobashi-Shimada-T.Y., hep-th/0209251

> To preserve the GKP-W relation in the PP-wave limit,
> we should consider
>
> ***'tunneling' geodesics***





Though this 'tunnuling' trajectory is apparently different from the null geodesic of the usual interpretation of the pp-wave limit, we have the same bulk 'Hamiltonian' with the ordinary Minkowski treatment:

At a purely *formal level*, [real picture $\longrightarrow$ tunneling picture] is equivalent to a 'double' Wick rotation:

◇ affine time : $\tau \longrightarrow -i\tau$

◇ target time : $t \longrightarrow -it$

$\Downarrow$

Tunneling geodesic :

$$z = \frac{1}{\cosh\tau}, \quad t = \tanh\tau$$

in terms of the Poincaré coordinate of *Euclideanized* AdS spacetime:

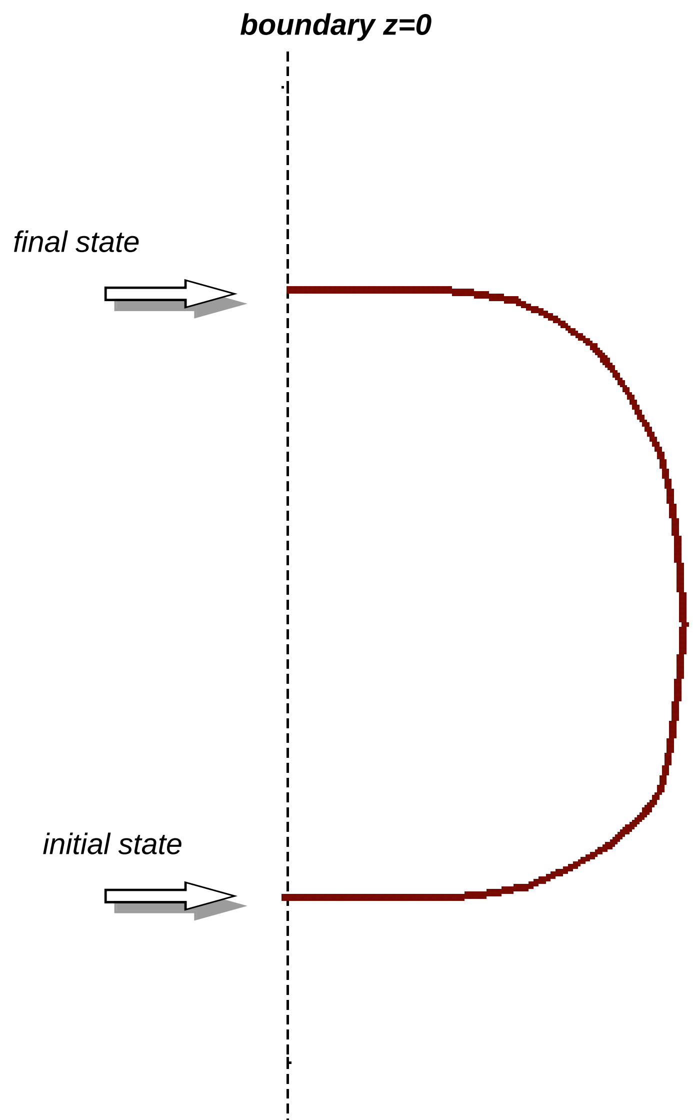
$$ds^2 = R^2(\frac{dz^2}{z^2} + \frac{dx_3^2 + dt^2}{z^2} + \cdots)$$

◇ Reaches the boundary $z \rightarrow 0$ as $\tau = \pm T, \quad T \rightarrow \infty$: $\quad z \rightarrow 2\mathrm{e}^{-T}$

This explains why

free 'asymptotic' Hamiltonian $\sim$ dilatation

is correct, by way of renormalization group, in spite of the fact that Hamiltonian *cannot* be directly identified with dilatation in our approach.



- Affine time direction near the boundary is manifestly orthogonal to the boundary. :

  τ should *not* be identified *directly* with the radial time

- boundary $\rightarrow$ boundary $\Leftrightarrow$ infinite affine time interval:

  $\tau = -T \rightarrow +T \quad T \rightarrow \infty$ (no periodicity)

- Boundary theory must be treated as Euclidean,

  because we are considering tunneling amplitudes

solves all of the 'puzzles'!

$\diamond$ observables = *Euclideanized* S-matrix elements [$\leftarrow$ bulk]

(*in the sense of* $1\,(\sim \tau) + 1\,(\sim \psi)$ *dimensions*)

$$\Updownarrow$$

$\diamond$ correlators of local gauge invariant BMN operators [$\leftarrow$ boundary]

Derivation from Witten diagrams

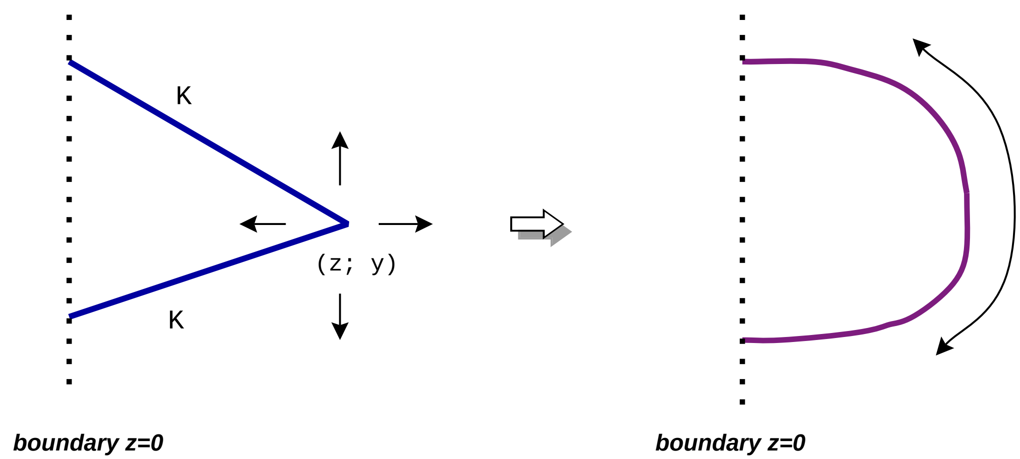
Boundary correlation functions are computed diagramatically by Witten diagrams using the bulk-to-boundary $(z, \vec{x}) \leftrightarrow (0, \vec{y})$ propagator.

$$K_\Delta(z, \vec{y}; \vec{x}) = \left(\frac{z}{z^2 + (x-y)^2}\right)^\Delta$$

- 2-pt function:

$$\frac{1}{|x-x'|^{2\Delta}} = \lim_{\epsilon \to 0} \frac{\epsilon}{N(\Delta)^2} \int \frac{d^4y dz}{z^5} z^\epsilon K_\Delta(z, \vec{y}; \vec{x}) K_\Delta(z, \vec{y}; \vec{x}')$$

In the limit of large $\Delta = J + k$ (k=finite), the integral is dominated by a one-dimensional integral along the tunneling null trajectory.



The tunneling trajectory connecting points from boundary to boundary (uniquely) solves the saddle-point eq.

$$\frac{\partial}{\partial z}\Big[\ln K_\Delta(z,\vec{y};\vec{x}) + \ln K_\Delta(z,\vec{y};\vec{x}')\Big] = 0,$$

$$\frac{\partial}{\partial y^\mu}\Big[\ln K_\Delta(z,\vec{y};\vec{x}) + \ln K_\Delta(z,\vec{y};\vec{x}')\Big] = 0$$

$$\Downarrow$$

$$z(\tau) = \frac{|x-x'|}{2\cosh\tau}, \quad y^\mu(\tau) = \frac{1}{2}(x+x')^\mu - \frac{1}{2}(x-x')^\mu \tanh\tau$$

$$\int \frac{d^4y\,dz}{z^5} z^\epsilon K_\Delta(z,\vec{y};\vec{x}) K_\Delta(z,\vec{y};\vec{x}') \sim \frac{N(\Delta)^2}{|x-x'|^{2\Delta}} \int_{-T}^{T} d\tau$$

– $\tau \sim$ “collective coordinate” with measure $\Leftrightarrow$ affine time along the tunneling trajectory

$$\frac{d^4y\,dz}{z^5} \longrightarrow d\tau$$

- We can extend this picture to 3-pt functions :

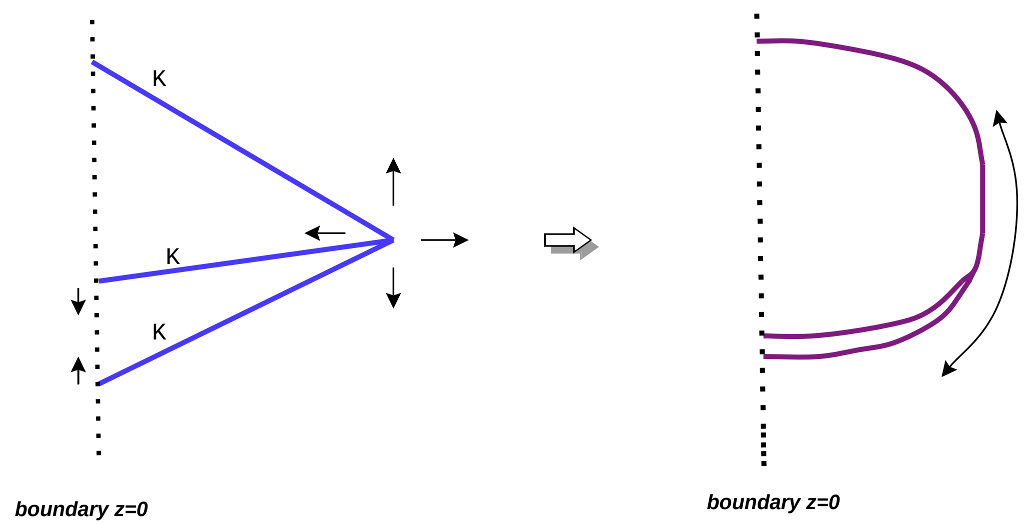
$$\frac{C_{123}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}|x_3-x_1|^{\Delta_3+\Delta_1-\Delta_2}}$$

$$=\frac{1}{N(\Delta_1,\Delta_2,\Delta_3)}\int\frac{d^4x dz}{z^5}K_{\Delta_1}(z,\vec{x};\vec{x}_1)K_{\Delta_2}(z,\vec{x};\vec{x}_2)K_{\Delta_3}(z,\vec{x};\vec{x}_3)V_{123}(z;\vec{x})$$

For generic configurations of 3 points x_1, x_2, x_3, there is no saddle point.

However, in an appropriate short-distance limit on SYM side, the integral is dominated by a single tunneling trajectory parametrized by the collective coordinate τ with appropriate cutoff.

$$\delta\sim|\vec{x}_2-\vec{x}_3|\rightarrow 0$$



$$\Downarrow$$

precise holographic relation between the 3-point vertex $V_{123} \sim \lambda_{123}$ and the CFT coefficient C_{123}.

For general stringy modes,

$$\langle \overline{O}_1 O_2 O_3 \rangle \sim \frac{\lambda_{123}}{|\vec{x}_1 - \vec{x}_c|^{\Delta_1+\Delta_2+\Delta_3}} \int_{-\infty}^{+\infty} d\tau \, \mathrm{e}^{-(\Delta_1-\Delta_2-\Delta_3)\tau} \; \exp[-f \frac{J_2 J_3}{J_1} \frac{(2\delta)^2}{|\vec{x}_1 - \vec{x}_c|^2} \mathrm{e}^{2\tau}]$$

$\Uparrow$ boundary-bulk propagators

$\Uparrow$ cutoff factor

- The factor f takes into account the nonlocal nature of string interactions:

$$f \frac{J_2 J_3}{J_1} \to \frac{J_1}{4\pi\mu|\alpha_{(1)}|} = \frac{R^2}{4\pi\alpha'} = g_s N \quad \text{for large } \mu$$

This leads to the Holographic relation for the OPE coefficients of BMN operators:

$$C_{123} = \frac{\tilde{\lambda}_{123}}{\mu(\Delta_2 + \Delta_3 - \Delta_1)}$$

$$\tilde{\lambda}_{123} = \left(f\frac{J_2 J_3}{J_1}\right)^{-(\Delta_2+\Delta_3-\Delta_1)/2} \Gamma(\frac{\Delta_2 + \Delta_3 - \Delta_1}{2} + 1)\lambda_{123}$$

$(f = 1 - 4\mu\alpha_{(1)}\alpha_{(2)}\alpha_{(3)}K \sim$ takes into account the extendedness of strings$)$

$$\lambda_{123} = {}_{(1)}\langle 1|_{(2)}\langle 2|_{(3)}\langle 3|\frac{\sqrt{J_1 J_2 J_3}}{N}\,|H_3\rangle_h,$$

Remarks:

- normalization of 2-point correlators

$$\langle O_1(\vec{x}_1) O_2(\vec{x}_2)\rangle = \frac{\delta_{12}}{|\vec{x}_{12}|^{2\Delta_1}}$$

- $|H_3\rangle_h$ is the 3-point interaction vertex of the 'holographic' string field theory

$$S_3 = \frac{1}{2}\int d\tau \;{}_{(1)}\langle\overline{\psi}|_{(2)}\langle\psi|_{(3)}\langle\psi|\frac{\sqrt{J_1 J_2 J_3}}{N}|H_3\rangle_h + h.c. \,.$$

- For the special case of impurity-preserving sector, where

$$\Delta_2 + \Delta_3 - \Delta_1 \sim O(1/\mu^2), \quad 1/\mu^2 \propto R^4/J^2$$

this reduces to the simple form $C \sim \lambda/(\Delta_2 + \Delta_3 - \Delta_1)$ in the limit of large μ, But this particular form is *only* valid with our unique *'holographic' string-field theory vertex.*

Our relation is valid for much more general case of impurity-non-preserving processes.

◇ Large-μ expansion : $\omega_n^{(r)} = \sqrt{(\mu\alpha_{(r)})^2 + n^2} \sim |\mu\alpha_{(r)}| + \frac{n^2}{2|\mu\alpha_{(r)}|} + \cdots$

$(\alpha_{(r)} = \alpha' p_r^+)$

$\sim$ large-α' expansion for fixed R^2/J

$\sim$ 'tensionless' limit

$\sim$ short-distance ('high-energy') limit

or for fixed $\alpha_{(r)}$ ($\sim$ string length parameter)

$\sim$ weak 't Hooft-coupling expansion on SYM side

$\sim$ small AdS-scale (large curvature) on bulk side

Thus in the large μ limit, we can compare both sides of AdS and CFT by perturbative methods.

Properties of the holographic string-field theory

- satisfyies the SUSY algebra to the first order in the string coupling
- The prefactor consists of two parts

$$|H_3\rangle_h = \frac{1}{2}(P_D + P_{SV})|E_3\rangle$$

$|E_3\rangle$ = standard overlap vertex of light-cone SFT

$P_D = (h^{(2)} + h^{(3)} - h^{(1)}) \sim$ energy ($h^{(r)}$) difference

Di Vecchia et al

$P_{SV} \sim$ Brink-Green-Schwarz type prefactor

Spradlin-Volovich, ...

Both of these prefactors have so far been discussed separately. However, our holographic relation demands that they must be combined with the *uniquely fixed* coefficients as above.

For purely bosonic states,

$$|H_3\rangle_h \Rightarrow \sum_{r=1}^{3}\left(\sum_{i=5}^{8}\sum_{m=0}^{\infty}\frac{\omega_m^{(r)}}{\alpha_{(r)}}a_m^{(r)i\dagger}a_m^{(r)i} + \sum_{i=1}^{4}\sum_{m=1}^{\infty}\frac{\omega_m^{(r)}}{\alpha_{(r)}}a_{-m}^{(r)i\dagger}a_{-m}^{(r)i}\right)|E_3\rangle$$

⇑ cos-modes
scalar directions ($\in S^5$)

⇑ sin-modes
vector directions ($\in$ AdS$_5$)

In particular, for bosonic supergravity modes (chiral primary states),

$$|H_3\rangle \Rightarrow (h_s^{(1)} + h_s^{(2)} - h_3^{(3)})|E_3\rangle$$

$$h_s^{(r)} = \mu\sum_{i=5}^{8} a_{i,0}^{(r)\dagger}a_{i,0}^{(r)} \quad \sim \quad \text{excitation number of only scalar directions}$$

◇ This corresponds to the known fact in supergravity that the derivative 3-point interaction terms can be eliminated by a field redefinition in the bulk of AdS_5 geometry :

Lee-Minwalla-Rangamani-Seiberg, hep-th/9806074.

After reducing to the single tunneling trajectory, this means that the derivative part of interaction vertex is contained in the form

$$V_{der} \sim (\frac{\partial}{\partial \tau} + h^{(1)} + h^{(2)} - h^{(3)})\phi^{(1)}\phi^{(2)}\overline{\phi}^{(3)} \sim 0 \quad \Leftarrow \text{eq. of motion}$$

with $h^{(i)}$ being the *total* free Hamiltonian.

Note: Total derivative cannot be ignored: *energies (as conformal dimensions) are not conserved* in our Euclideanized S-matrix picture. The existence of the P_D part is crucial for the holographic relation.

Explicit checks in the large-μ limit

- Impurity-preserving case:
  We have checked our predictions by direct computation for two-impurity cases with general vector and fermionic excitations, to the leading order in the large-μ expansion.

  We can also use previously known results for 3-point functions and explain the compatibility of *seemingly contradictory results* in the literature.

  - It is known that at the first order in λ' the stringy single-trace operators are not operators with *definite* conformal dimensions. To obtain operators with *definite* conformal dimensions, mixing with double-trace operators with degenerate (in the lowest $(\lambda')^0$ order) conformal dimensions must be taken into account.

    Beisert et. al., hep-th/0208178, Constable et. al., hep-th/0209002

    Let the order $g_2\lambda'$ part of the two-point functions for $\overline{O}_1$ and normal-product (local) operator $:O_2O_3:$ be $\qquad (J_1 = J_2 + J_3)$

$$\langle \overline{O}_1 : O_2O_3 :\rangle_{g_2\lambda'} = g_2\lambda'(\Delta_2^{\langle 1\rangle} + \Delta_3^{\langle 1\rangle} + b_{123})C_{123}^{\langle 0\rangle} \log(x_{12}\Lambda)^{-2}$$

$$\lambda'\Delta_r^{\langle 1\rangle} = \text{first order anomalous dimension}$$

The operators must then be replaced by

$$O_{1,single} \to O'_1 \equiv O_{1,single} + g_2 D_{123} : O_2 O_3 :$$

$$D_{123} = \frac{\Delta_2^{\langle 1 \rangle} + \Delta_3^{\langle 1 \rangle} - \Delta_1^{\langle 1 \rangle} + b_{123}}{\Delta_2^{\langle 1 \rangle} + \Delta_3^{\langle 1 \rangle} - \Delta_1^{\langle 1 \rangle}} C_{123}^{\langle 0 \rangle}$$

– Then, 3-point correlator is

$$\langle \overline{O}'_1 O'_2 O'_3 \rangle = C_{123} \times \text{standard spacetime factors} + \text{higher orders}$$

$$C_{123} = -g_2 \lambda' \frac{b_{123} C_{123}^{\langle 0 \rangle}}{\Delta_2 + \Delta_3 - \Delta_1} \sim O(g_2)$$

$$\Uparrow$$

$$\Delta_2 + \Delta_3 - \Delta_1 \sim O(\lambda')$$

so that the correct 3-point OPE coefficient to the leading order in the large-μ expansion is this C_{123}, not $g_2 C_{123}^{\langle 0 \rangle}$

– On the other hand, it has also been proposed that the *'wrong'* coefficient $C^{\langle 0\rangle}_{123}$ satisfies (in the large-μ limit)

$$\frac{\sqrt{J_1 J_2 J_3}}{N}\langle 1,2,3|\frac{1}{2}P_D|E_3\rangle = g_2\mu(\Delta_2+\Delta_3-\Delta_1)C^{\langle 0\rangle}_{123}$$

Di Vecchia et. al., hep-th/0304025

– Our prediction for the impurity-preserving case

$$C_{123} = \frac{\sqrt{J_1 J_2 J_3}}{N}\langle 1,2,3|\frac{1}{2\mu}(P_D+P_{SV})|E_3\rangle + \text{higher orders with respec to } 1/\mu$$

allows us to compute the matrix elements of the vertex $P_{SV}|E_3\rangle$, using these two observations.

The results explain the known proposal which relates the mixing matrix of SYM in *a particular basis* to $P_{SV}|E_3\rangle$

Gomis-Moriyama-Park, hep-th/0210153, hep-th/0301250

Previously, it was not clear how to justify their special choice of the basis.

All these discussions has been restricted to impurity preserving sector.

- Impurity non-preserving case:

  This case has not been treated before at least in the context of the BMN limit.

  – When impurities are not preserved, the correction factor

$$\Big(f\frac{J_2J_3}{J_1}\Big)^{-\frac{(\Delta_2+\Delta_3-\Delta_1)}{2}}\Gamma(\frac{\Delta_2+\Delta_3-\Delta_1}{2}+1)$$

  in our holographic relation plays a crucial role, since in the large-μ limit

$$\Big(f\frac{J_2J_3}{J_1}\Big)^{-\frac{(\Delta_2+\Delta_3-\Delta_1)}{2}} \sim \Big(\frac{J_1}{4\pi\mu\alpha_{(1)}}\Big)^{-\frac{(\Delta_2+\Delta_3-\Delta_1)}{2}} = (g_{\rm YM}^2N)^{-\frac{(\Delta_2+\Delta_3-\Delta_1)}{2}}$$

  in explaining the correct order of 3-point correlation functions.

  – Also, the Gamma-function factor could be singular when $\frac{\Delta_2+\Delta_3-\Delta_1}{2} \sim$ negative integer in the *classical* approximation.

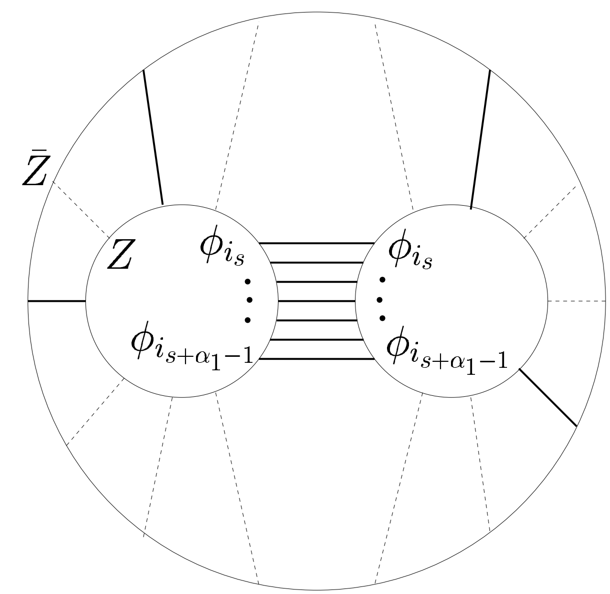
$$\frac{\Gamma(\frac{\Delta_2+\Delta_3-\Delta_1}{2}+1)}{\frac{\Delta_2+\Delta_3-\Delta_1}{2}} = \sum_{n=0}^{\infty}\frac{2(-1)^n/n!}{\Delta_2+\Delta_3-\Delta_1+2n} + \text{nonsigular terms}$$

– Exchanges of arbitrary number of scalars with non-singlet representation (SO(4)) can be treated in a general way:

◇ Outer circle = 1
◇ Two inner cirlces = 2 and 3

dotted lines : Z-$\overline{Z}$ contractions

solid lines : ϕ-ϕ contractions



– Non-singlet vector exchanges can also be treated in a general way inductively.

Mixings with double-trace operators must be taken into account in this case, when there is no propagators between 2 and 3.

– Exchanges of singlet impurities :

This case is somewhat subtle and more interesting.

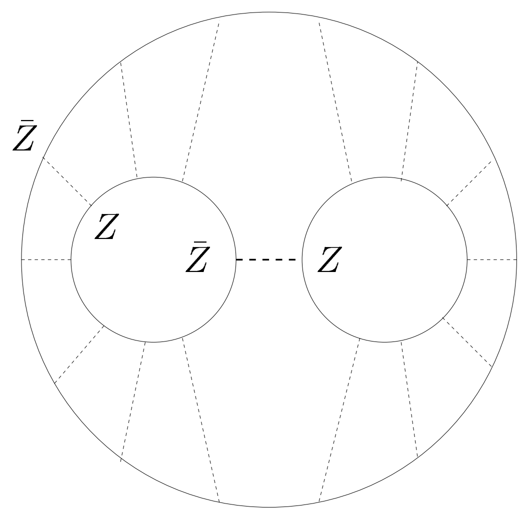
Consider two simple but typical examples:

1. Scalar singlet operator with $\Delta_2 + \Delta_3 - \Delta_1 > 0$

$$\overline{O}^{(1)} = \frac{1}{\sqrt{J_1 N^{J_1}}} \mathrm{Tr}[\overline{Z}^{J_1}], \quad O^{(3)} = \frac{1}{\sqrt{J_3 N^{J_3}}} \mathrm{Tr}[Z^{J_3}]\,,$$

$$O^{(2)}_{s(p;-p)} = \frac{1}{2\sqrt{J_2 N^{J_2+2}}} \mathrm{Tr}\Big[\sum_{a=0}^{J_2} \phi_i Z^a \phi_i Z^{J_2-a}\, \mathrm{e}^{-\frac{2\pi i}{J_2} ap} - 4\overline{Z} Z^{J_2+1}\Big]\,.$$

The leading contribution comes from the following diagram with a Z-$\overline{Z}$ exchange



The CFT coefficient is simply

$$C_{123} = -2\frac{\sqrt{J_1 J_2 J_3}}{N}\frac{1}{J_2}\,.$$

On the string side, the matrix element of the interaction vertex is
($\tilde{N}^{rs}_{mn}$ = Neumann coefficient)

$$4\mu\frac{\sqrt{J_1 J_2 J_3}}{N}(-\tilde{N}^{22}_{p,-p}) = -4\mu\frac{\sqrt{J_1 J_2 J_3}}{N}\frac{1}{4\pi\mu|\alpha_{(1)}|y}$$

The multiplying factor in the holographic relation is

$$\frac{1}{\mu(\Delta_2+\Delta_3-\Delta_1)}\Big(f\frac{J_1 J_2}{J_3}\Big)^{-1}\Gamma(\frac{\Delta_2+\Delta_3-\Delta_1}{2}+1) \sim \frac{1}{2\mu}\frac{4\pi\mu|\alpha_{(1)}|}{J_1}$$

This preciesely gives the above CFT coefficient.

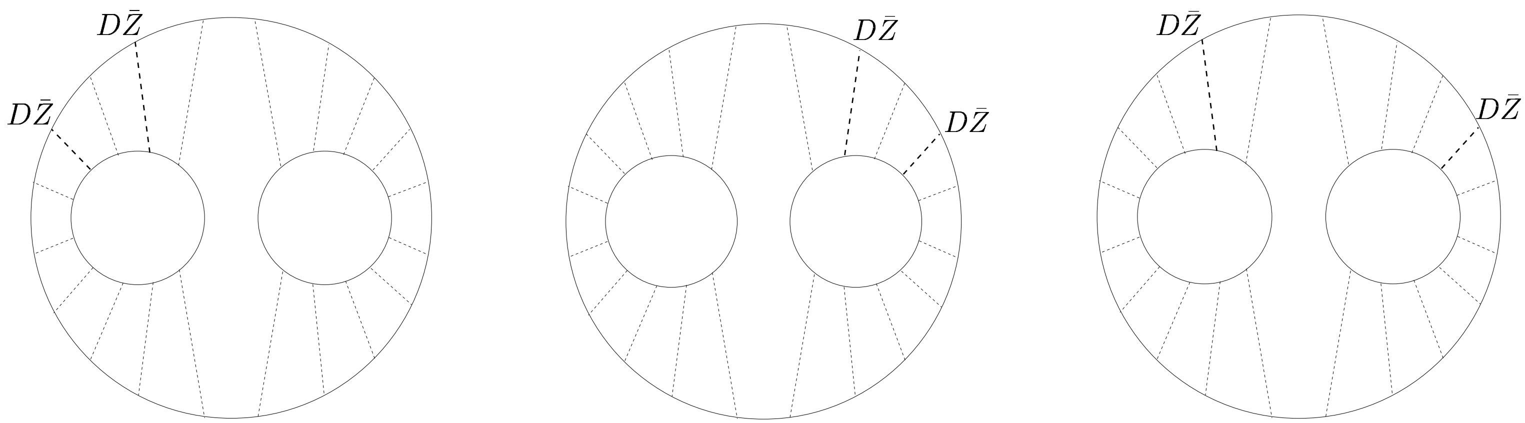
(notation: $y = J_2/J_1, 1-y = J_3/J_1$)

2. Singlet vector operator with $\Delta_2 + \Delta_3 - \Delta_1 < 0$

$$\overline{O}^{(1)}_{vs(p;-p)} = \frac{1}{4\sqrt{J_1 N^{J_1}}} \sum_{a=0}^{J_1-2} \mathrm{Tr}\left[(D_j\overline{Z})\overline{Z}^a (D_j\overline{Z})\overline{Z}^{J_1-2-a} \mathrm{e}^{\frac{2\pi i}{J_1-1}ap}\right]$$

with $O^{(2)}$ and $O^{(3)}$ are both ground-state operators.

The correlation function on SYM side consists of 3 different types of Feynman diagrams.



Each contribution is

$$F_{123}^{1\leftrightarrow 2} = \frac{2}{N\sqrt{J_1J_2J_3}}J_2J_3 \times J_1^2 \frac{\sin^2 \pi y p}{(\pi p)^2}\frac{1}{|\vec{x}_{12}|^{2J_2+2}|\vec{x}_{13}|^{2J_3}},$$

$$F_{123}^{1\leftrightarrow 3} = \frac{2}{N\sqrt{J_1J_2J_3}}J_2J_3 \times J_1^2 \frac{\sin^2 \pi(1-y)p}{(\pi p)^2}\frac{1}{|\vec{x}_{12}|^{2J_2}|\vec{x}_{13}|^{2J_3+2}},$$

$$F_{123}^{1\leftrightarrow (2,3)} = \frac{4}{N\sqrt{J_1J_2J_3}}J_2J_3 \times J_1^2(-1)^p \frac{\sin \pi y p \sin \pi(1-y)p}{(\pi p)^2}\frac{(\vec{x}_{12}\cdot\vec{x}_{13})}{|\vec{x}_{12}|^{2J_2+2}|\vec{x}_{13}|^{2J_3+2}}.$$

This leads to the CFT coefficient

$$C_{123} = 2\frac{\sqrt{J_1J_2J_3}}{N}J_1\frac{\sin^2 \pi y p}{(\pi p)^2}$$

as the coefficient in front of the correct spacetime factor.

On the string side, the matrix element of the interaction vertex consists of 3 terms

$$-4\frac{\sqrt{J_1J_2J_3}}{N}\frac{\mu}{2}\left(-2\tilde{N}^{11}_{p,-p}+\tilde{N}^{11}_{-p,-p}+\tilde{N}^{11}_{pp}\right)=\frac{\sqrt{J_1J_2J_3}}{N}\frac{8}{\pi|\alpha_{(1)}|}\sin^2(\pi\mu p)$$

The multiplying factor for this case is

$$\frac{1}{\mu}f\frac{J_2J_3}{J_1}\frac{(\mu\alpha_{(1)})^2}{p^2}=\frac{1}{\mu}\frac{J_1}{4\pi\mu|\alpha_{(1)}|}\frac{(\mu\alpha_{(1)})^2}{p^2}=\frac{J_1|\alpha_{(1)}|}{4\pi p^2}$$

by which we find again precise matching.

The following specific characteristics of our holographic string-field theory and of the holographic relation, respectively, are crucial for the exact matching.

- Prefactor:
  scalar $\leftrightarrow$ $\cos$-modes only   vector $\leftrightarrow$ $\sin$-modes only
- Multiplying factor:
  depends on the sign of $\Delta_2+\Delta_3-\Delta_1$, singular (for $p=0$) in the latter case.

Concluding Remarks

We have

◇ proposed a natural spacetime picture for holography in the BMN limit

◇ proposed a precise and general holographic relation for OPE coefficients [boundary] and 3-point string vertex [bulk]

◇ checked the matching of bulk and boundary to the leading order in perturbation theory in $1/\mu$

- Remaining problems
  - higher orders both in g_s and λ'
  - higher-point functions
  - extension to more general operators (spin chain, ...)
  - .....
  - derivation of the holographic relation within the logic of SYM $\leftrightarrow$ *origin of fifth dimension ?*
  - extension to nonconformal case of general Dp-brane backgrounds
    see Asano-Sekino-T.Y. hep-th/0308024
  - extension to M-theory holography, AdS$_{4(7)} \times S^{7(4)}$ ...

- Extension to Dp-brane backgrounds :

  Our picture does not depends on exact conformal symmetry, and hence can be applied to the plane-wave limit of a more general Dp-brane backgrounds ($p \neq 3$).

  Since the tunneling trajectories do not go into the singular horizons, we can in principle use the same strategy as for $p = 3$.

  Even at the level of two-point functions for sugra modes, we have quite nontrivial predictions.

  Asano-Sekino-T.Y. hep-th/0308024

  For example, the two-point functions for scalar excitations behave as $(p < 5)$

$$|\vec{x}_1 - \vec{x}_2|^{-\frac{4}{5-p}J + c(p)}, \quad c(p) = J\text{-independent const.}$$

  – We can extract effective spacetime dimensions for scalar excitations from this result:

$$d_{eff}(p) = 2 + \frac{4}{5-p}, \quad \Delta_{eff} = \frac{2}{5-p}$$

  $\Rightarrow$ integer value effective spacetime dimensions when $p = 1, 3, 4$

$$d_{eff}(1) = 3, \quad d_{eff}(3) = 4, \quad d_{eff}(4) = 6,$$

This is consistent with the possible existence of nontrivial conformal field theories (with maximal susy) related to AdS holography (such as $\mathrm{AdS}_4 \times S^7$) in M-theory ($d_{eff} = 3 \sim$ M2 and $d_{eff} = 6 \sim$M5 branes).

($p = 1 \rightarrow$) 2D SYM with $g_{\mathrm{YM}} \rightarrow 0, \quad g_{\mathrm{YM}}^2 N \rightarrow \infty$

$\Updownarrow$

Matrix-string theory in the weak($g_{\mathrm{YM}} \sim 0$)-coupling limit

(Recall perturbative string theory $\Leftrightarrow$ strong-coupling limit)

$\Updownarrow$ Sekino-T.Y. hep-th/0108176, T.Y. hep-th/0210243

(Wrapped) supermembrane
in the decompactification limit $R_{11} \sim g_s(\sim 1/g_{\mathrm{YM}}^2) \rightarrow \infty$

$\updownarrow$?

3D conformal field theory

Scale dimension of scalar fields $= 1/2$ for CFT ($\mathrm{AdS}_4 \times S^7$ case)

Scales for AdS$_4 \times S^7$

- 11 D Plack scale: $\ell_{11} = g_s^{1/3} \ell_s$
- Compactification radius: $R_{11} = g_s \ell_s$
- D2 brane scale: $L_{\rm D2} = g_{\rm YM}^{-2} \sim g_s^{-1} \ell_s$
- M2 scale (scale of AdS$_4$ with flux N): $L_{\rm AdS} = \ell_{11} N^{1/6}$

◇ M-theory limit: $R_{11} \to \infty$ keeping ℓ_{11} fixed $i.\ e.\ g_s \to \infty, \ell_s \to 0$

$$\Downarrow$$

$$L_{\rm D2} \sim \ell_s^4 / \ell_{11}^3 \to 0$$

◇ Expected entropy:

$$S \propto L_{\rm AdS}^9 \propto N^{3/2}$$

The boundary CFT dual to AdS$_4 \times S^7$is an extreme *IR limit of ($\mathcal{N} = 8$) 3D SYM*, but *its degrees of freedom must be of order* $N^{3/2}$ (not naive N^2) in the large N limit.

No lagrangian description ? (for a related discussion, see J. H. Schwarz, hep-th/0411077))