



# Cosmic Strings In SUSY-Breaking Meta-Stable Vacua



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hep-th/0610042

# 1. Realistic solitons?



1. Motivation: **Growing expectation of observing Cosmic Strings**

Gravitational wave detectors and WMAP data provide possible observation of solitons in the universe!

2. Arena : **Recently found “SUSY-breaking Meta-stable Vacua”**

Intrilligator, Seiberg and Shih found a simple mechanism of dynamical SUSY-breaking, in low energy superQCD's!

3. Tool : **Brane-Engineering of Solitons**

D-brane construction of field theory solitons is so powerful that one can predict various solitons from string theory!

**1+2+3 → Our result : Cosmic strings in SUSY breaking vacua**

# Plan of this talk

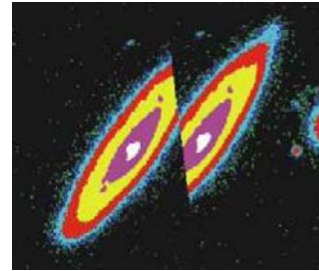
- ☑ 1. Motivation + Arena + Tool 1 page
  
- ▶ 2. Three streams:
  - 2-1) Cosmic strings and their observation 2 pages
  - 2-2) ISS model and its SUSY-breaking vacua 2 pages
  - 2-3) Brane engineering of solitons 3 pages
  
- ▶ 3. **Solitons in ISS vacua**
  - 3-1) Brane realization of ISS vacua
  - 3-2) **Predictions for solitons: brane-engineering**
  - 3-3) **Explicit solutions verifying the predictions**
  
- ▶ 4. Concluding words

## 2-1) Cosmic Strings and their Observation

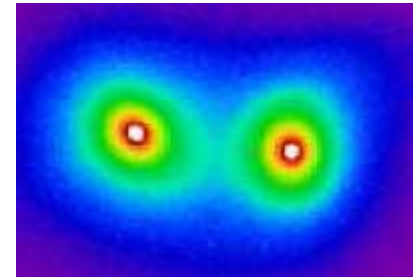
**Cosmic strings** = Macroscopic 1-dim. objects in cosmic space

Observation expected:

(a) **Direct observation**:  
Gravitational Lensing  
by the cosmic strings.



astro-ph/0611744



CSL-1 ×

(b) **Gravitational waves**: kinks on the Nambu-Goto strings travelling with the speed of light emit grav. waves.

Constraint from observation  $\longrightarrow G_N \cdot \mu < 10^{-6}$

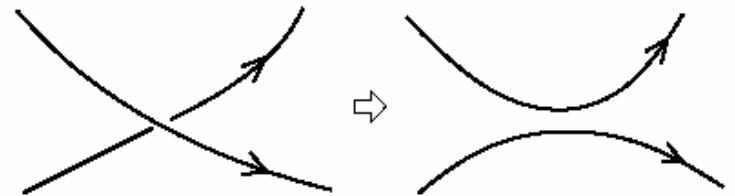
What are they made of ?: **Vortex solitons in field theories**

- String-like objects are co-dimension 2: vortices in 1+2 dim.
- Associated with spontaneous U(1) symmetry breaking.  
(Abelian Higgs model is often used in cosmology)

## Generation and Evolution

Kibble mechanism generates cosmic strings in thermal cooling of the universe with GUT symmetry breaking.

No monopole-like problem:  
“reconnection” of colliding strings  
reduces the number.



Number density is correlated with reconnection probability  $P$ .

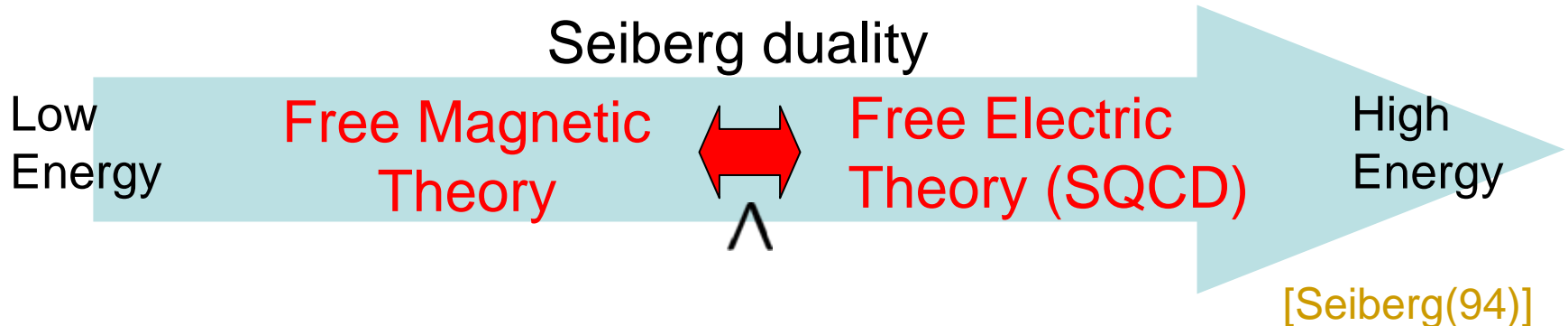
However, this  $P$  is difficult to compute..... Because  
no analytic solution for vortex strings  
→ Moduli space undetermined  
→ dynamics unknown.....

We'll see a power of the Brane-Engineering to compute  $P=1$

Note: cosmic superstrings/D-branes have  $P < 1$

## 2-2) ISS Model and SUSY-Breaking Vacua

Surprisingly, simple super QCDs have meta-stable vacua which break supersymmetries dynamically!



For  $N_c + 1 \leq N_f \leq \frac{3}{2}N_c$ , low energy region is well-described by a **Seiberg-dual** (“magnetic”) theory with a gauge group  $SU(N)$  ( $N = N_f - N_c$ ). The matter consists of **dual quarks** and a **meson**, with a superpotential

$$W = h\text{Tr}_c [qM\tilde{q}] - h\mu^2\text{Tr}_f M$$

( $\mu^2$  : quark mass)

	$SU(N)$	$SU(N_f)_V$
$M_{[N_f \times N_f]}$	1	adj.
$q_{[N \times N_f]}$	$\square$	$\bar{\square}$
$\tilde{q}_{[N_f \times N]}$	$\bar{\square}$	$\square$

Perturbative vacua of the magnetic theory at low energy

$$\mathcal{L} = \text{Tr}_c \left[ -\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} + \mathcal{D}_\mu q \mathcal{D}^\mu q^\dagger + \mathcal{D}_\mu \tilde{q}^\dagger \mathcal{D}^\mu \tilde{q} \right] + \text{Tr}_f [\partial_\mu M^\dagger \partial^\mu M] - V$$

$$V_F = |h^2| \text{Tr}_f \left[ |\tilde{q}q - \mu^2 \mathbf{1}_{N_f}|^2 \right] + |h^2| \text{Tr}_c \left[ |qM|^2 + |\tilde{q}^\dagger M^\dagger|^2 \right]$$

$$V_D = \frac{g^2}{4} \text{Tr}_c \left[ (qq^\dagger - \tilde{q}^\dagger \tilde{q})^2 \right] - \frac{g^2}{8} (\text{Tr}_c qq^\dagger - \text{Tr}_c \tilde{q}^\dagger \tilde{q})^2.$$

The rank condition  $\tilde{q}q = \mu^2 \mathbf{1}_{N_f}$  cannot be satisfied



**Supersymmetry is spontaneously broken!**

[Intrilligator, Seiberg, Shih(06)]

Property of the vacua:

$$M = \begin{pmatrix} 0_{N \times N} & 0_{N \times (N_f - N)} \\ 0_{(N_f - N) \times N} & M_0 \end{pmatrix} \quad \frac{q^\dagger}{\mu^*} = \frac{\tilde{q}}{\mu} = \begin{pmatrix} \mathbf{1}_N \\ 0_{(N_f - N) \times N} \end{pmatrix}$$

1. Meta-stable and long-lived (decaying to susy vacua)

2. Cosmological constant  $V = |h\mu^2|^2 (N_f - N) > 0$

3. Moduli space of vacua:  $\mathcal{M}_{\text{vac}} = \frac{U(N_f)}{SU(N) \times U(N_f - N)}$

## 2-3) Brane-Engineering of Solitons

Solitons in field theories can be realized by brane configurations.

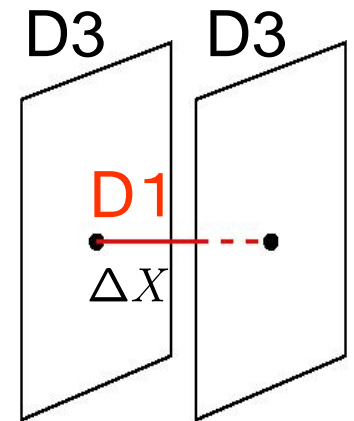
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- Geometrical understanding → prediction for new solitons
  - New ways to look at them → derivation of new properties

Ex) 'tHooft-Polyakov monopoles in SU(2) YM  
= D1 suspended between parallel D3s

(1) charge: End points of D1 carry magnetic charge on D3-branes

(2) susy: preserving half of 16.

(3) mass:  $\mathcal{T}_{D3}(2\pi\alpha')^2 = 1/g_{YM}^2$   
 $\Delta X = 2\pi\alpha'\phi_4 |_{r \rightarrow \infty}$  }  $m_{\text{monopole}} = \mathcal{T}_{D1}\Delta X$

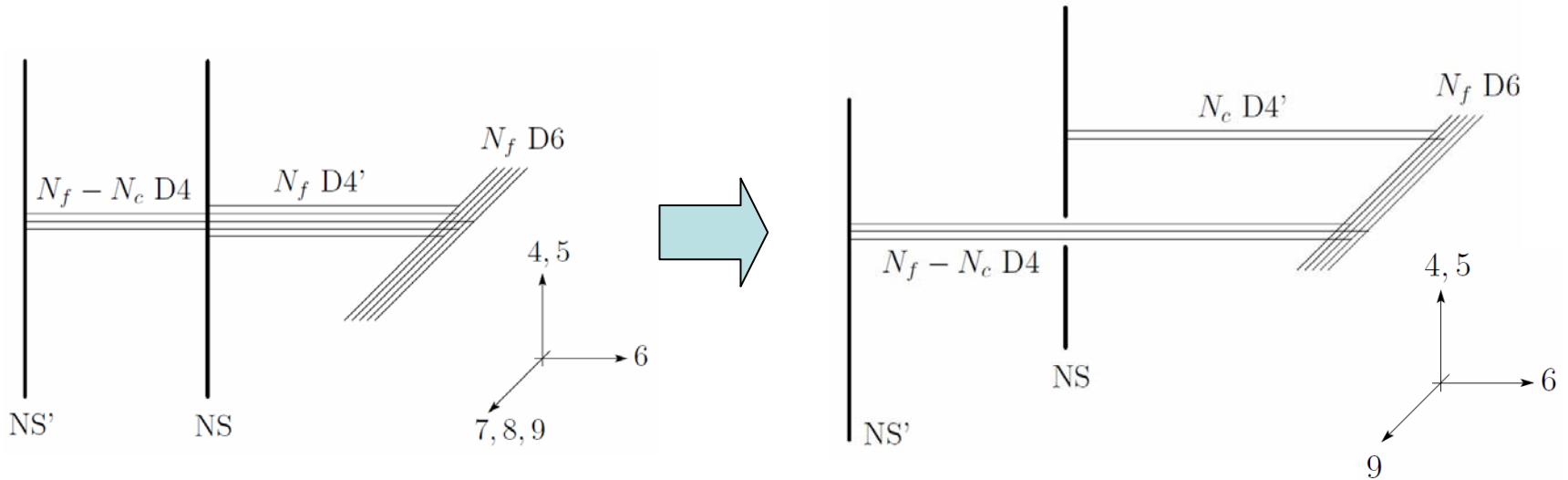


(4) Nahm eq.(Moduli space) = BPS eq. in D1 effective theory

(5) Nahm construction = pair annihilation of D3-D3bar

[Douglas] [Diaconescu] [Terashima, KH(05)]

# Ex) vortex strings in nonAbelian-Higgs model



Brane realization of  
 $N=2$   $U(N_f - N_c)$  SYM  
 w/ massless  $N_f$  fund.  
 matter (quarks)

[Hanany, Witten]

$$N = N_f - N_c$$

FI term added:  
 nonAbelian Higgs model

$$W = g \text{Tr}_c [q \Sigma \tilde{q}] - v^2 \text{Tr}_c \Sigma$$

$$V_F = \text{Tr}_c \left\{ g^2 |q \tilde{q} - v^2 \mathbf{1}_N|^2 + |q \Sigma|^2 + |\tilde{q}^\dagger \Sigma^\dagger|^2 \right\}$$

$$V_D = \text{Tr}_c \left\{ \frac{g^2}{4} (qq^\dagger - \tilde{q}^\dagger \tilde{q})^2 - \frac{1}{g^2} [\Sigma, \Sigma^\dagger]^2 \right\}$$

Vacua:  $\Sigma = 0, \quad qq^\dagger - \tilde{q}^\dagger \tilde{q} = 0, \quad q \tilde{q} = v^2 \mathbf{1}_N$

Vortex solution coming from BPS equations:

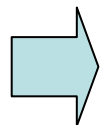
$$(D_1 \pm iD_2)\phi = 0, \quad F_{12} \pm \frac{g^2}{2} (2v^2 - \phi\phi^\dagger) = 0$$

$$q = \tilde{q}^\dagger \equiv \frac{\phi}{\sqrt{2}}, \quad \Sigma = A_0 = A_3 = 0$$

Energy saturation:  $\mathcal{E} = \mp 2v^2 \int d^2x F_{12} = 4v^2 \pi |k|$

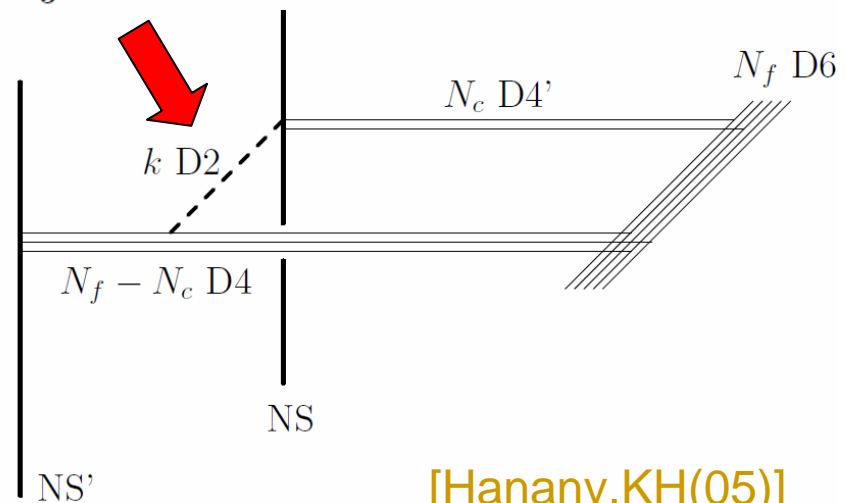
Brane realization:

$k$  D2's suspended between the NS5 and the D4's  
(charge, susy, mass: identical)



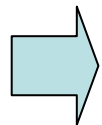
Moduli space is predicted

[Hanany, Tong(03)]



[Hanany, KH(05)]

[Tong, KH(05)]



- Reconnection probability  $P=1$  is obtained
- Moduli sp. verified in field theory (Moduli matrix formalism)
- Reconnection probability verified in field theory

[Eto, Marmorini, Nitta, Ohashi, Vinci, KH(06)]

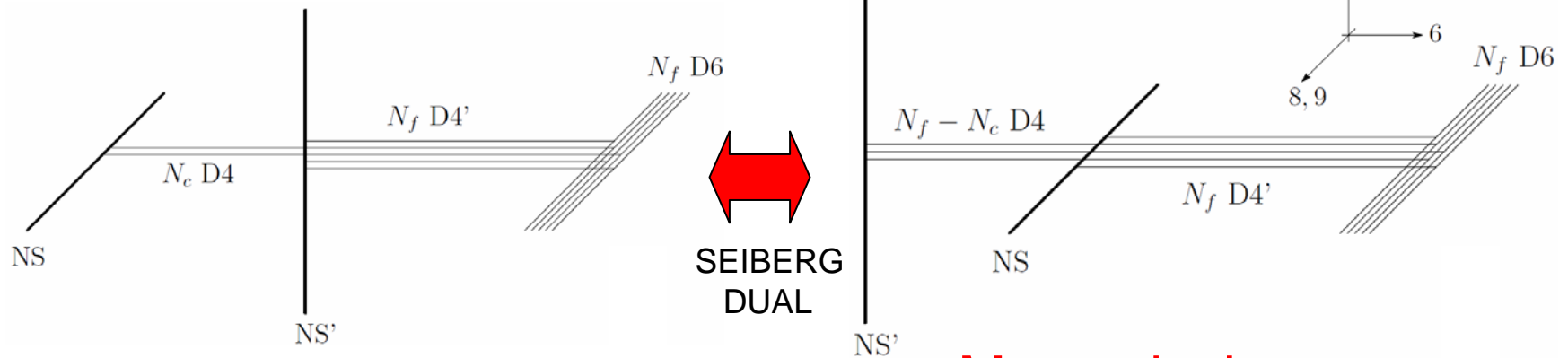
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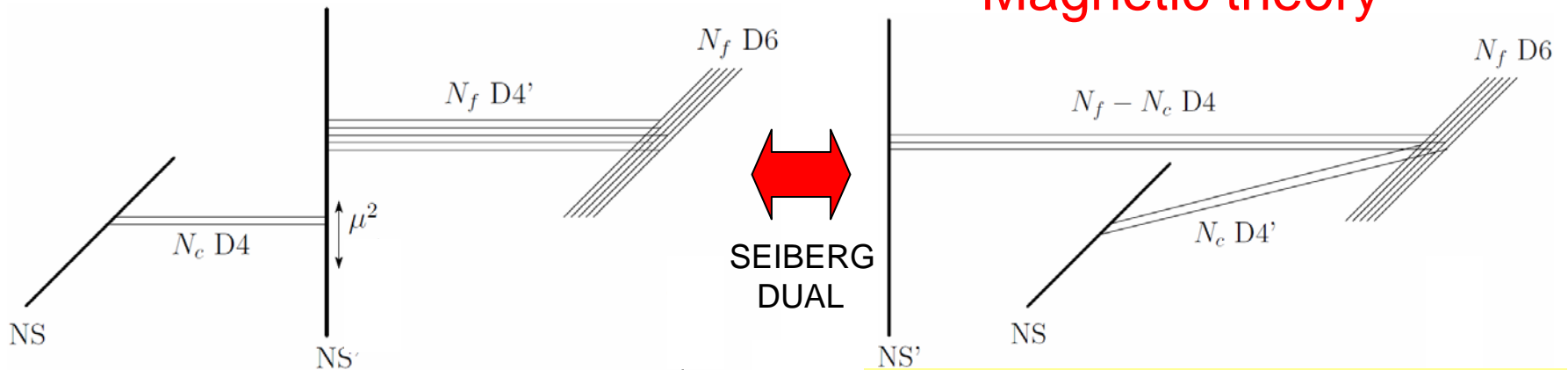
# 3-1) Brane-Realization of the ISS Vacua

[Ooguri, Ookouchi] [Franco, Garcia-Etxebarria, Uranga]

## N=1 Massless SQCD (ele.)



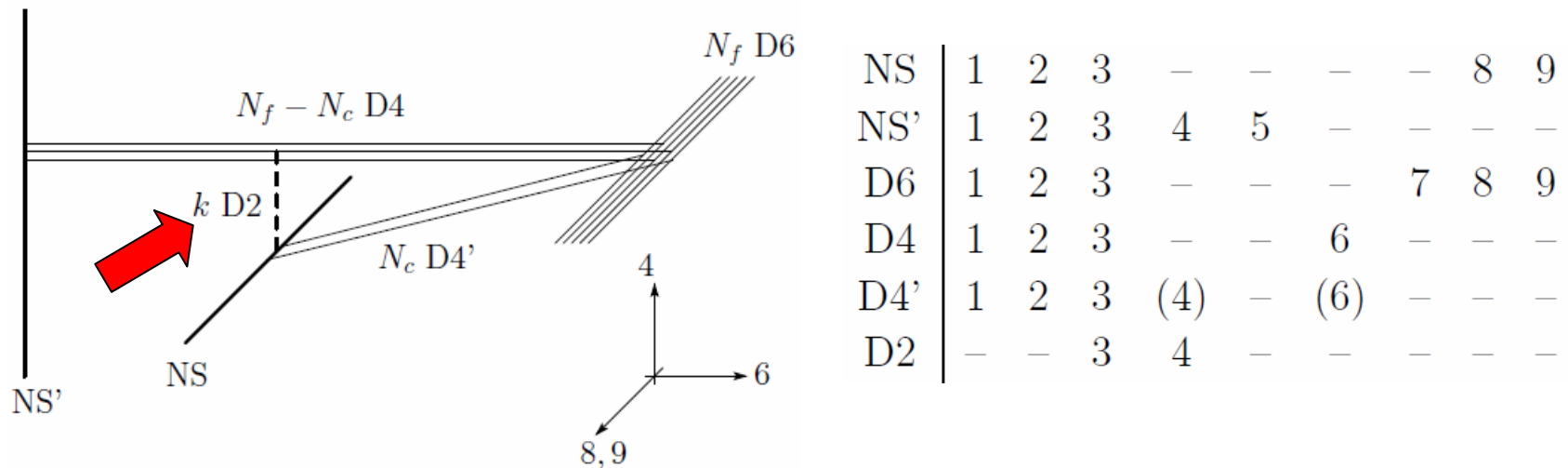
**Magnetic theory**



## N=1 Massive SQCD (ele.)

SUSY is broken by slanted D4, giving the c.c.

## 3-2) Prediction of Solitons: Brane-Engineering



D2 suspended between the D4 and the NS5 should be the vortex string, a la Hanany-Tong!

Prediction 1: tension of the string is proportional to  $\mu^2$

The length of the D2 should tell the tension of the vortex.

Prediction 2: the vortex breaks SUSY irrespective with the cosmological constant. The D2 and D6 break bulk SUSY completely, even without the tilted D4.



### 3-3) Explicit Solutions in Field Theory

We can verify the predictions by finding explicit solutions.

Ansatz:  $M = 0, \quad q = \tilde{q}^\dagger \equiv \frac{\phi}{\sqrt{2}}$

$$\tilde{\mathcal{L}} = \text{Tr}_c \left[ -\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} + \mathcal{D}_\mu \phi \mathcal{D}^\mu \phi^\dagger \right] - \frac{|h^2|}{4} \text{Tr}_f \left[ (\phi^\dagger \phi - 2\mu^2 \mathbf{1}_{N_f})^2 \right]$$

The potential term can be re-written as

$$\tilde{V} = |h^2 \mu^4| (N_f - N) + \frac{|h^2|}{4} \text{Tr}_c \left[ (\phi \phi^\dagger - 2\mu^2 \mathbf{1}_N)^2 \right]$$

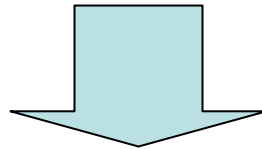
➡ Equivalence to well-known **nonAbelian semilocal strings!**  
(... But with a cosmological constant term.)

Embedding of Abelian semilocal strings is a solution,

$$F_{12} = \begin{pmatrix} 0 & & & \\ & \ddots & & \\ & & 0 & \\ & & & F_{12}^* \end{pmatrix}, \quad \phi = \begin{pmatrix} \sqrt{2}\mu & & & \\ & \ddots & & \\ & & \sqrt{2}\mu & \\ & & & \phi_0^* \quad \phi_1^* \quad \cdots \quad \phi_{N_f-N}^* \end{pmatrix}$$

## Verification

- 1) Tension of semilocal string : known to be proportional to  $\mu^2$
- 2,4) SUSY is broken irrespective with the C.C. term, since gaugino transformation  $\delta_\epsilon \lambda = \sigma^{\mu\nu} \epsilon F_{\mu\nu} + i\epsilon D$  does not vanish due to  $D \sim qq^\dagger - \tilde{q}^\dagger \tilde{q} = 0$
- 3) Multi-tension string exists, since we may change the entry of the embedding of the Abelian string solution.  
(This is consistent with the brane picture :  
End point of the D2 selects a D4 brane in the bunch)



All predictions from D-brane configurations are proven

Note: We need to have U(N) rather than SU(N),  
for the vortex solution to exist.

## 4. Concluding words



### Power of D-brane technique

The “Brane-Engineering” worked, as expected, to study solitons in new systems.

Existence and properties of solitons are predicted in string theory, and verified in field theory.

This works even for **non-BPS solitons**.

How far we can apply?

Ex) Stability of the soliton?

### Realistic cosmic strings?

Our cosmic strings reconnect with  $P=1$ .

Phenomenological models with the ISS mechanism?

[Kitano,Ooguri,Ookouchi(06)] [Murayama,Nomura(06)]

[Csaki,Shirman,Terning(06)] [Aharony,Seiberg(06)]

## Other solitons in the ISS vacua?

When U(1) is not gauged,

$$\mathcal{M}_{\text{vac}} = \frac{U(N_f)}{SU(N) \times U(N_f - N)} \implies \pi_i(\mathcal{M}_{\text{vac}}) = 0$$

**No soliton**

When U(1) is gauged,

$$\widetilde{\mathcal{M}}_{\text{vac}} = \frac{SU(N_f)}{S[U(N) \times U(N_f - N)]}$$

$$\implies \dots \pi_2(\widetilde{\mathcal{M}}_{\text{vac}}) = \mathbf{Z}$$

Sigma model lump string (= semilocal string)

Global monopole

$$\dots \pi_0(\widetilde{\mathcal{M}}_{\text{vac}}) = \pi_1(\widetilde{\mathcal{M}}_{\text{vac}}) = 0$$

No other global / sigma model solitons

**Only cosmic strings are relevant!**