宇宙論と超弦理論

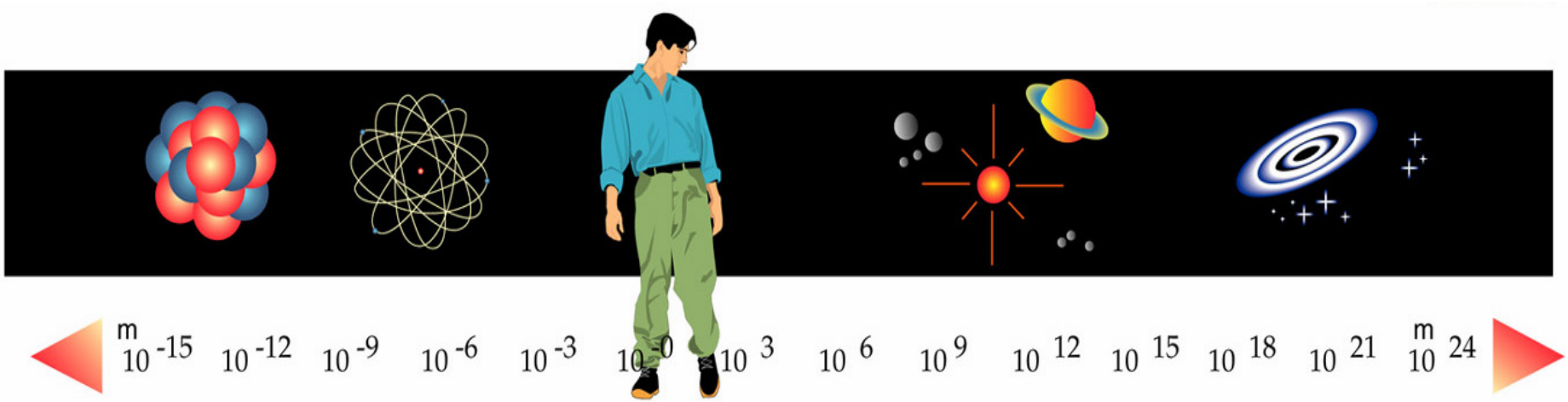
向山　信治（東大理）

弦理論研究会　＠　立教大学

Contents of this talk

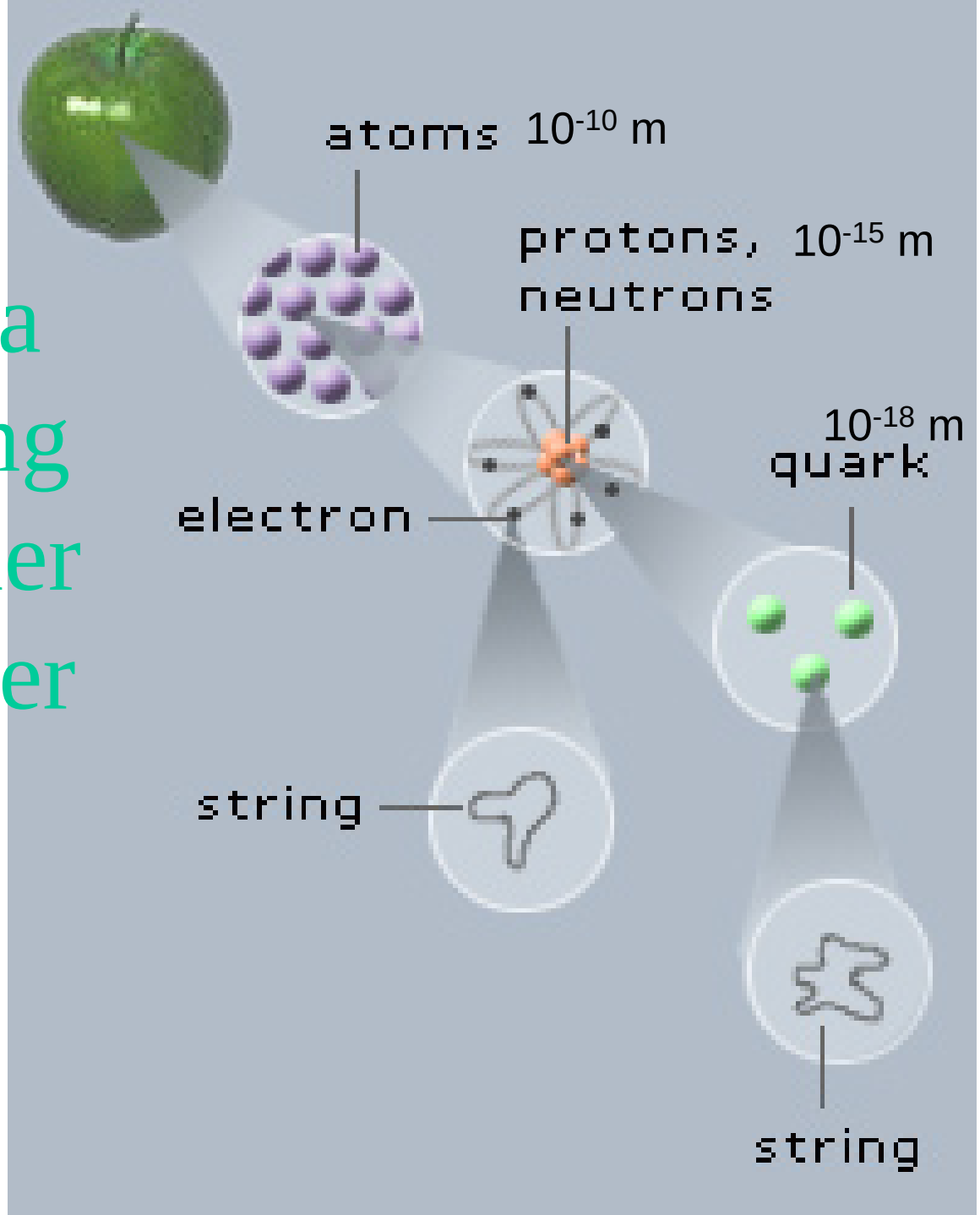
- Introduction
- Application I: Dark Matter & Chaotic Inflation
- Application II: “Fast-roll” inflation
- Application III: Higgs phase of gravity
- Summary

There are Frontiers in Physics:



at Short and Long Scales

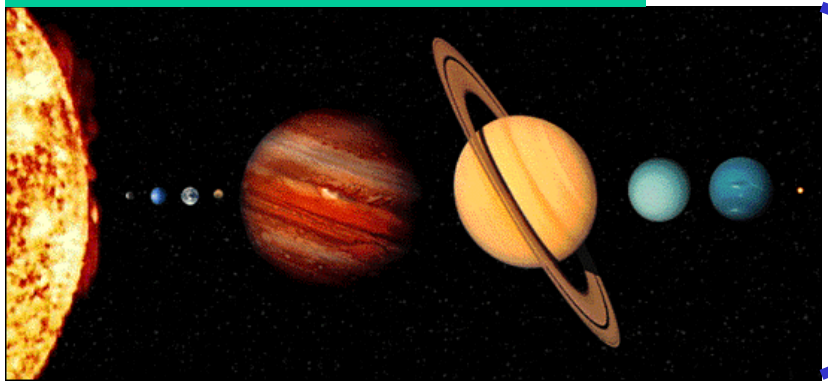
There is a story going into smaller and smaller scales.

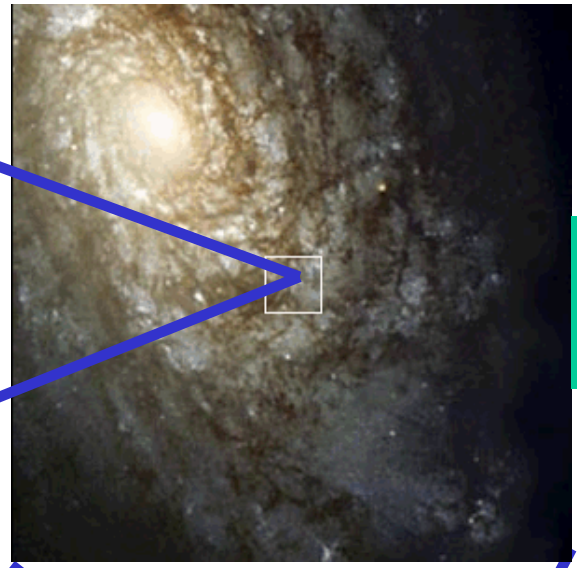


Also toward Larger scales

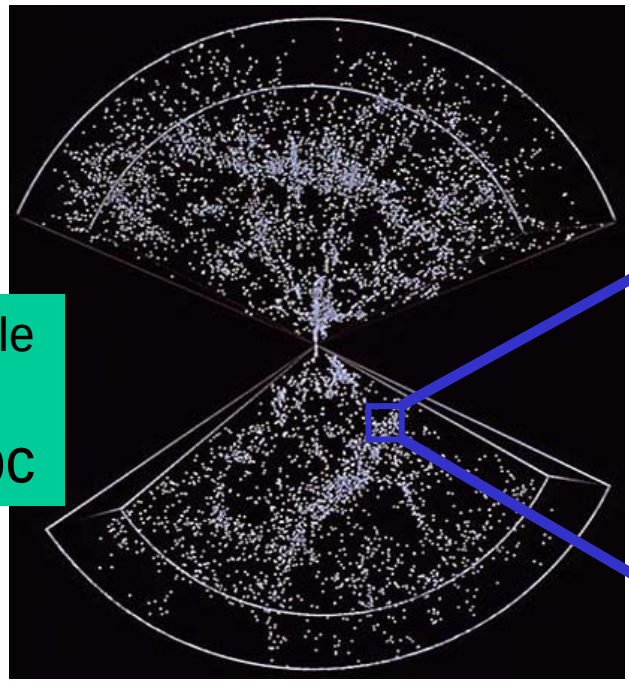
(pc = 3.3 light year=3.1×10^{18} cm)

Solar system 10^{15}cm

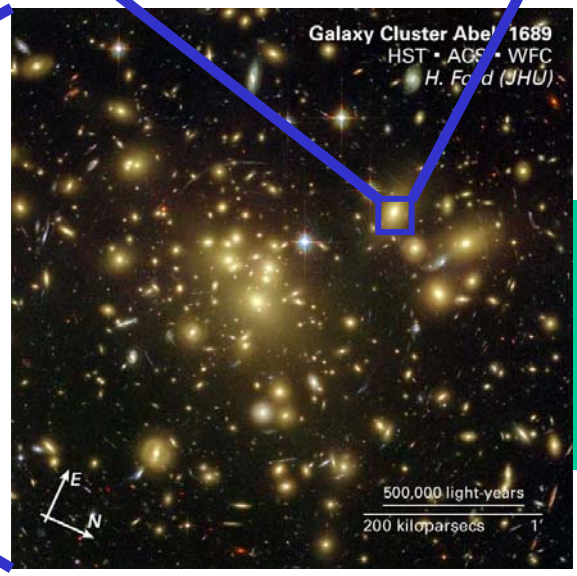




Galaxy
10 kpc



Large scale
structure
100 Mpc

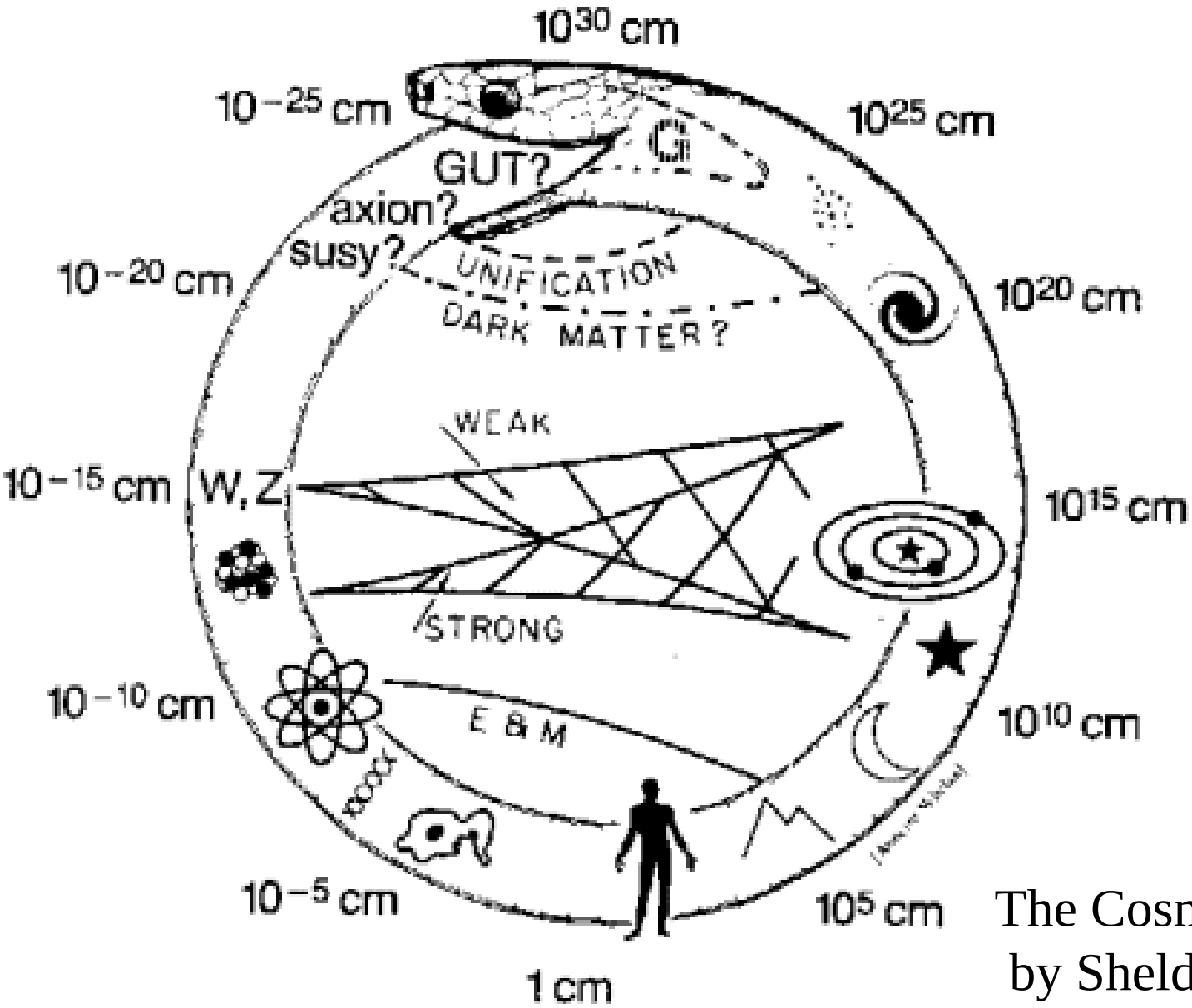


Cluster
of
galaxies
Mpc

At largest scale:

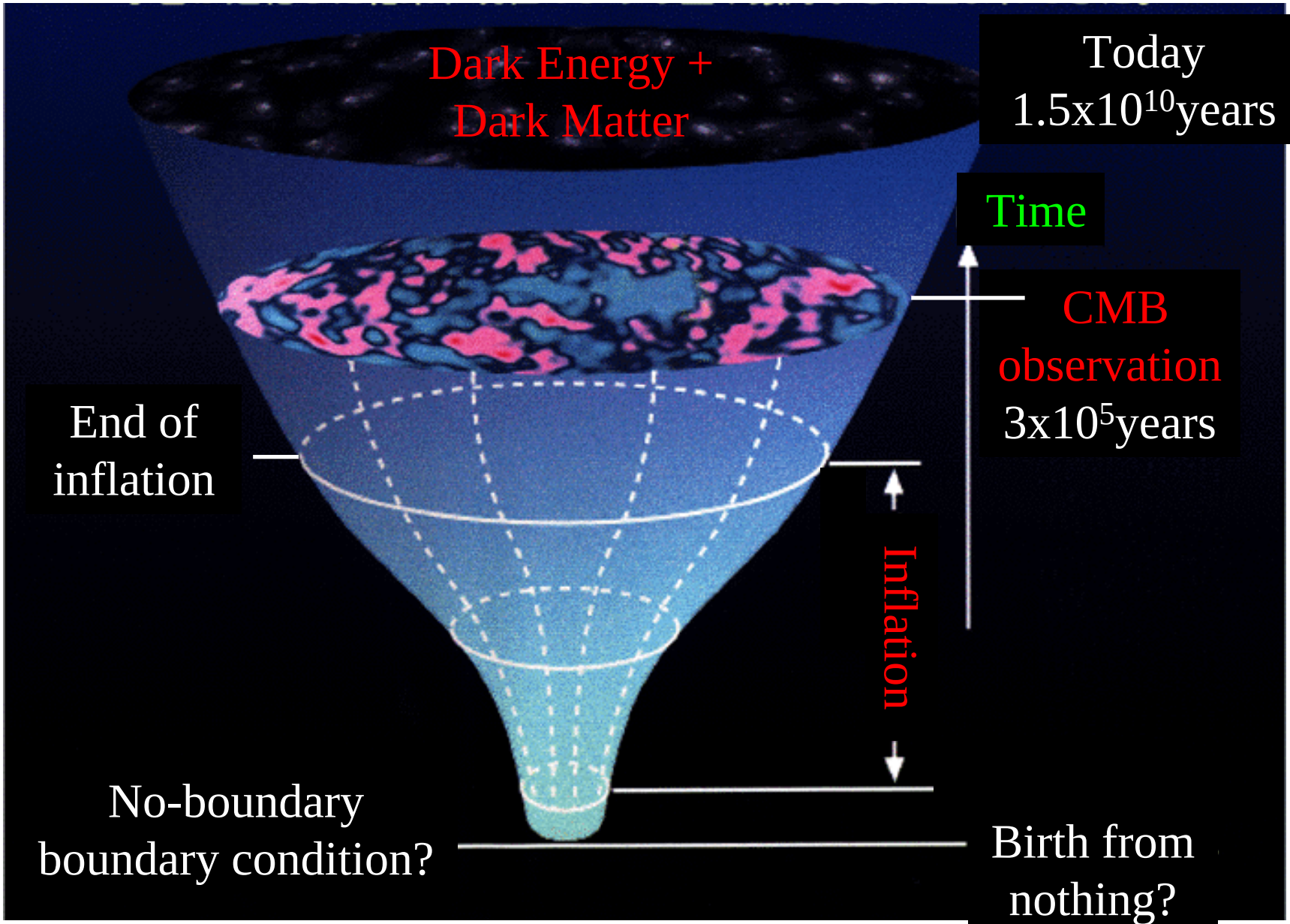
Cosmology

But...the largest scale and shortest scale must be connected!

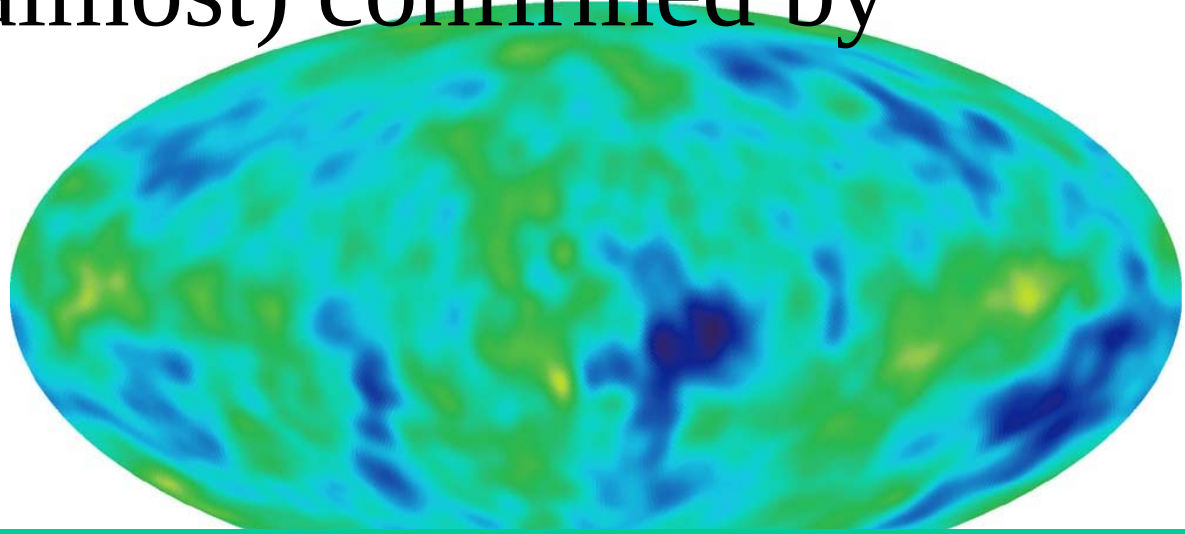


The Cosmic Uroboros
by Sheldon Glashow

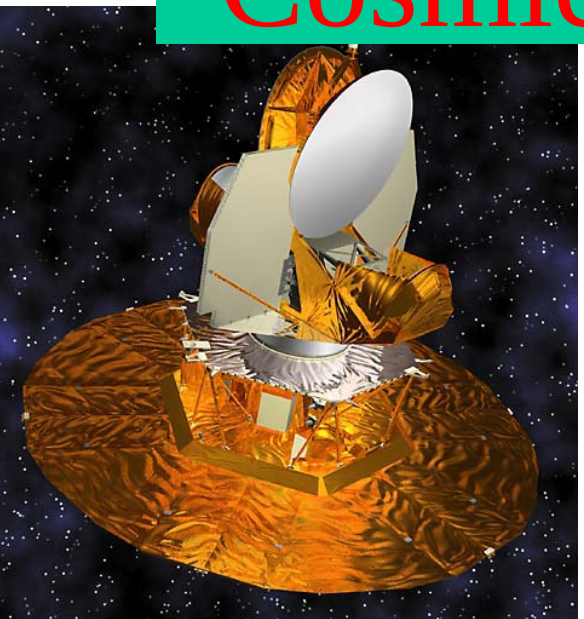
History of the Universe

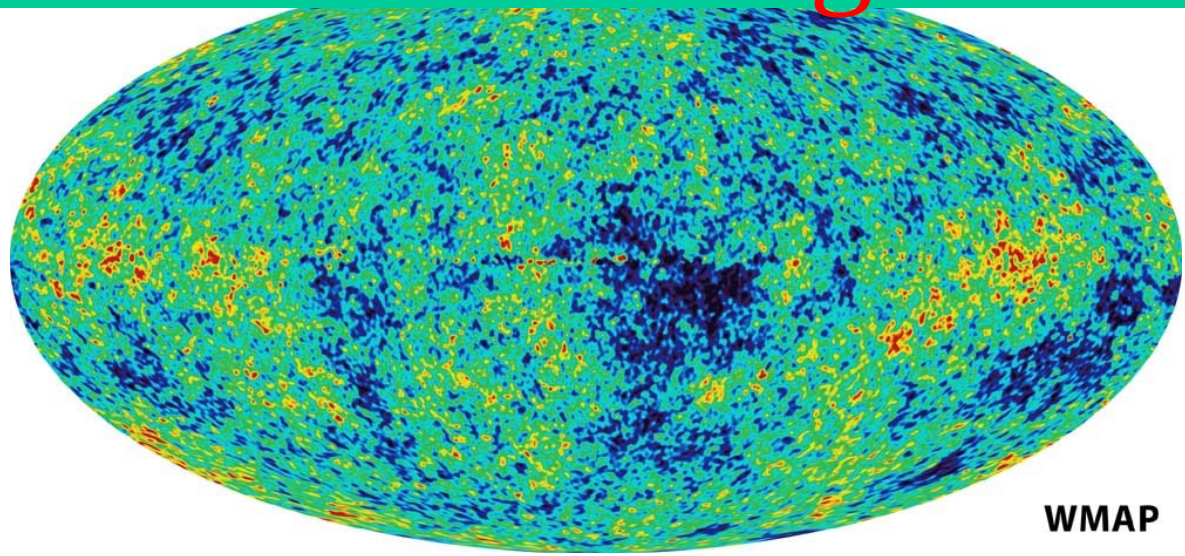


Inflation, dark energy and dark matter are (almost) confirmed by



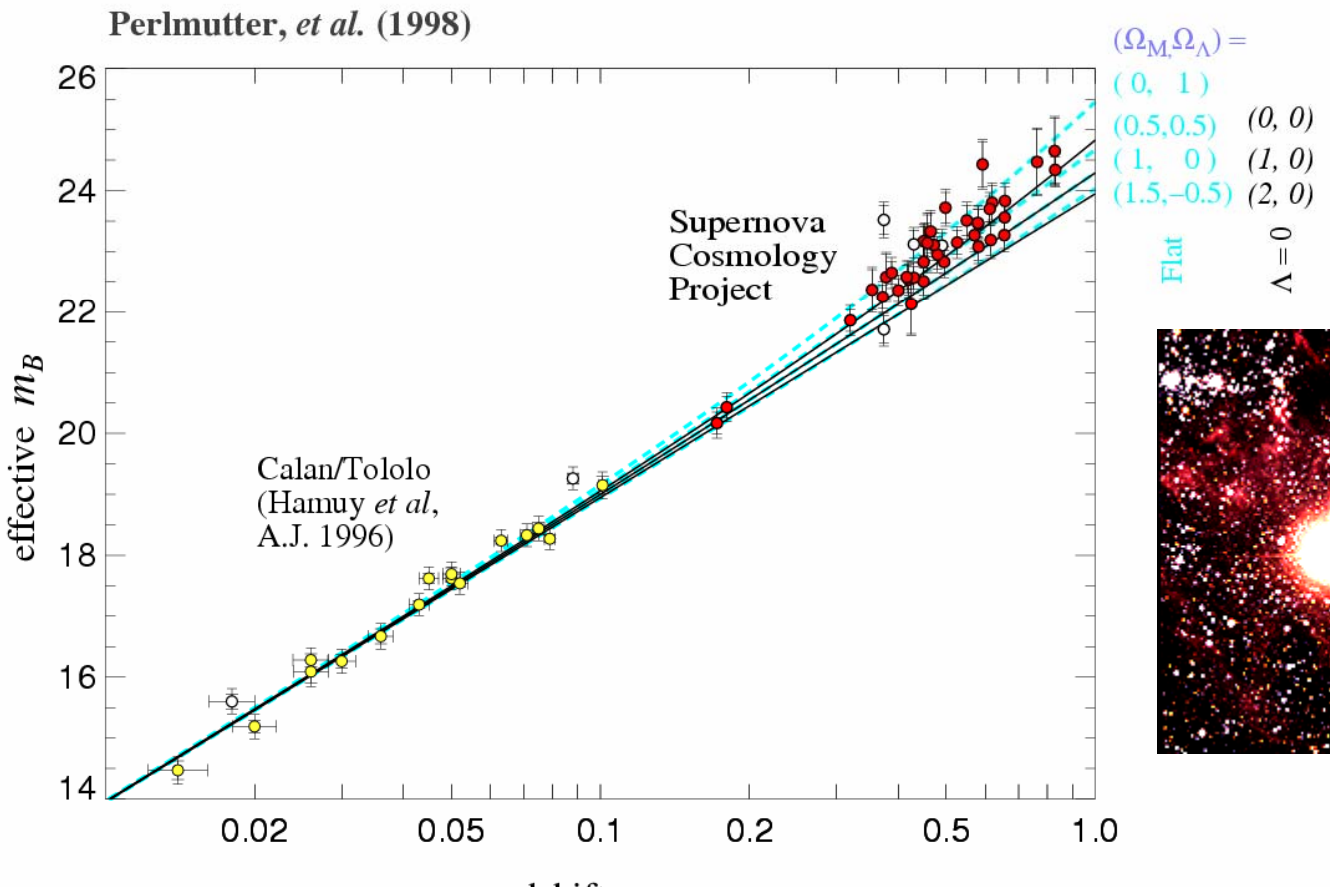
Cosmic microwave background

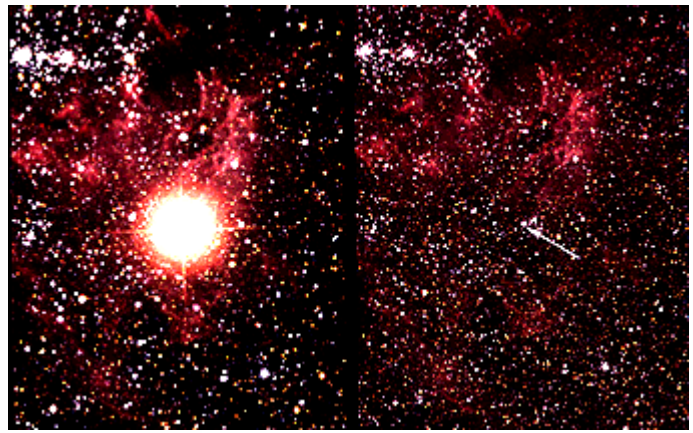




& Supernova observation

FAINTER (Farther) (Further back in time)





http://supernova.lbl.gov/

MORE REDSHIFT
(More total expansion of universe since the supernova explosion)

Three major mysteries in modern cosmlogy

- Early Universe
  Inflation
- Universe Today
  Dark Energy & Dark Matter

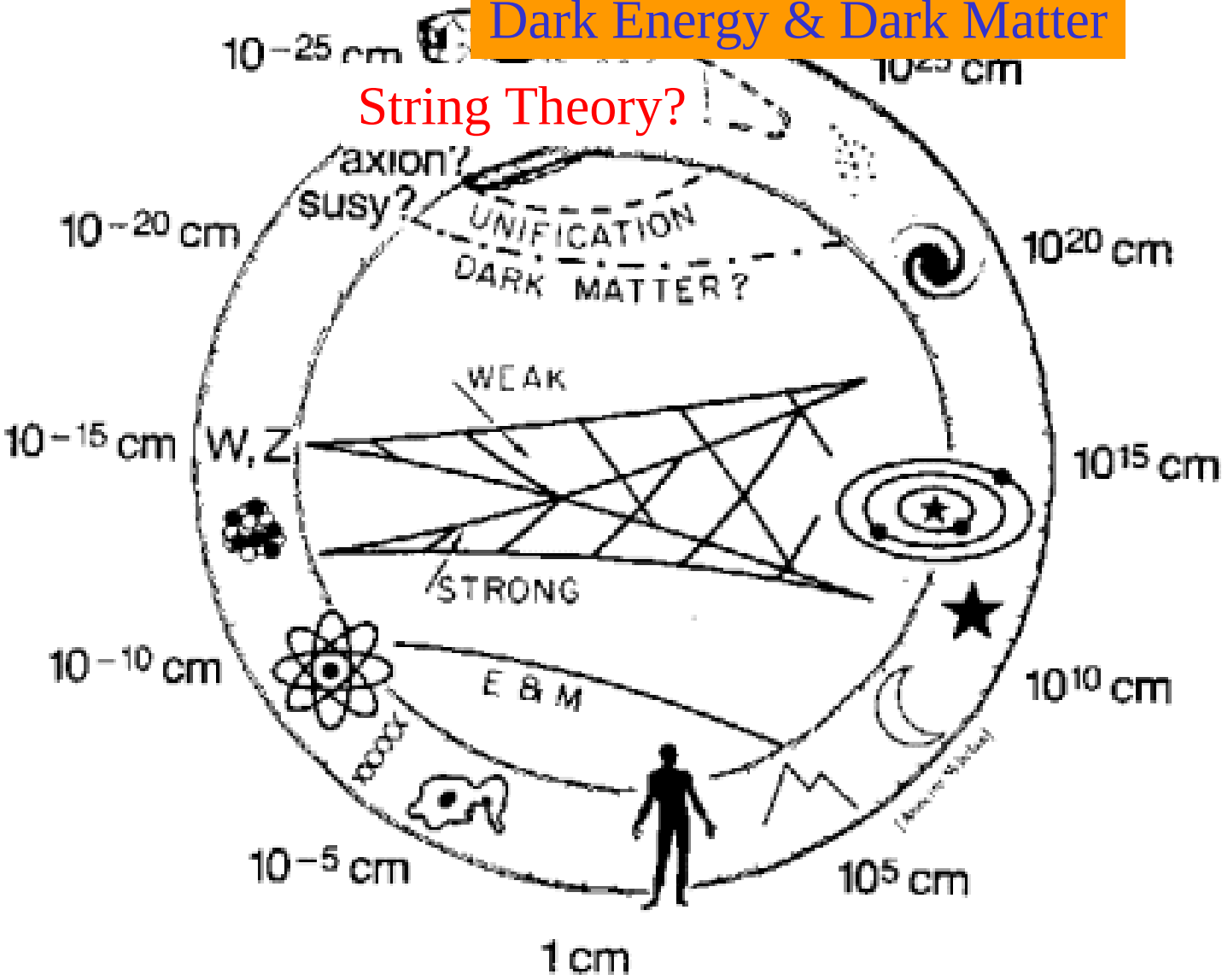
Two major (quasi-) de Sitter phases

We know they are (or were) there...

But, we don't know what they are.

Three mysteries: Inflation, Dark Energy & Dark Matter

String Theory?

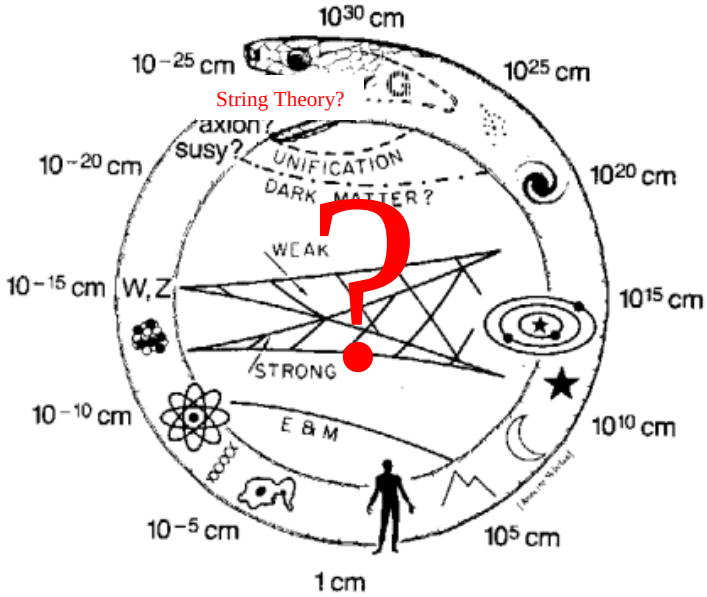


The Cosmic Uroboros by Sheldon Glashow

String theory until 2002

Bad thing

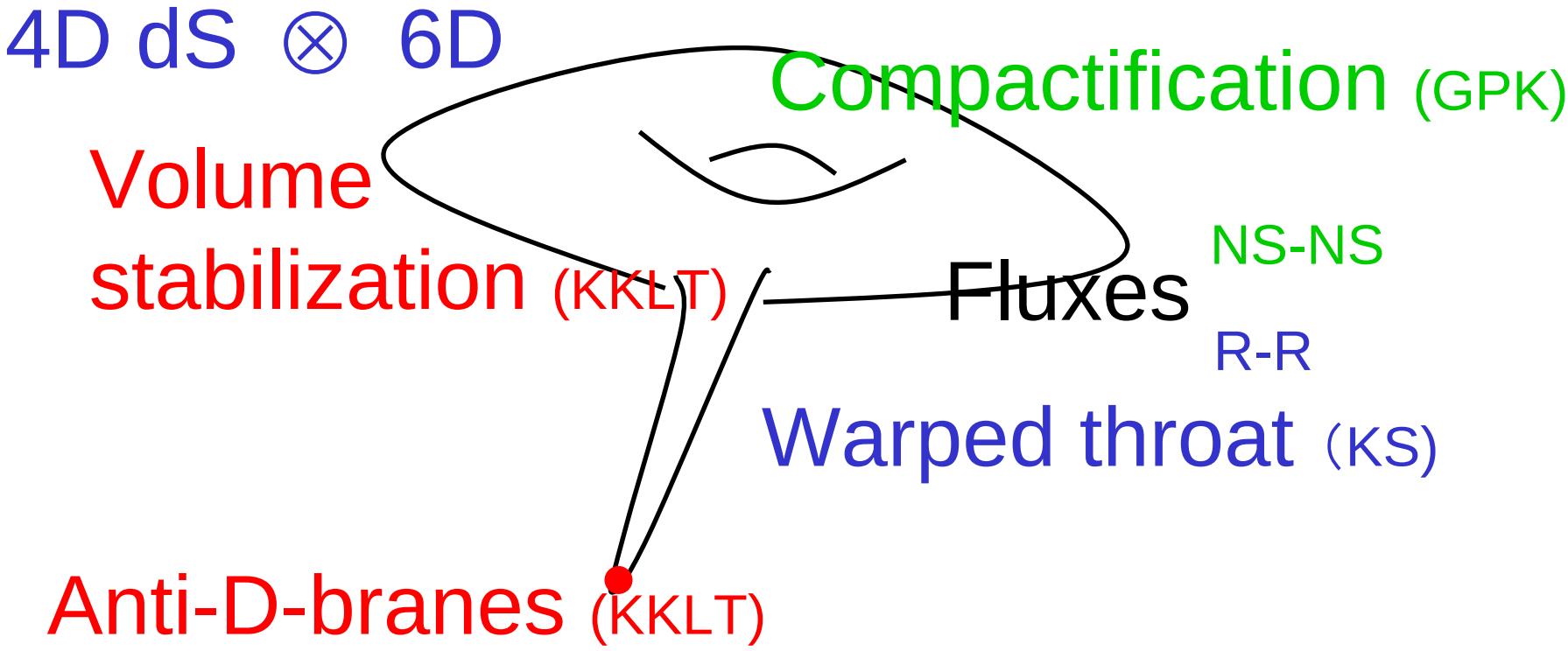
- No 4-dimensional de Sitter solution with stabilized moduli.
- No-go theorem!
- Contradict with inflation and dark energy?
- No way to reconcile with cosmology???



The Cosmic Uroboros does not close?

Recent Progress

- In 2003, a 4-dimensional de Sitter solution was finally found! Kachru, Kallosh, Linde and Trivedi (KKLT)
- In the previous no-go theorem, non-perturbative effects and branes were not taken into account.



Application I: Dark Matter & Chaotic Inflation

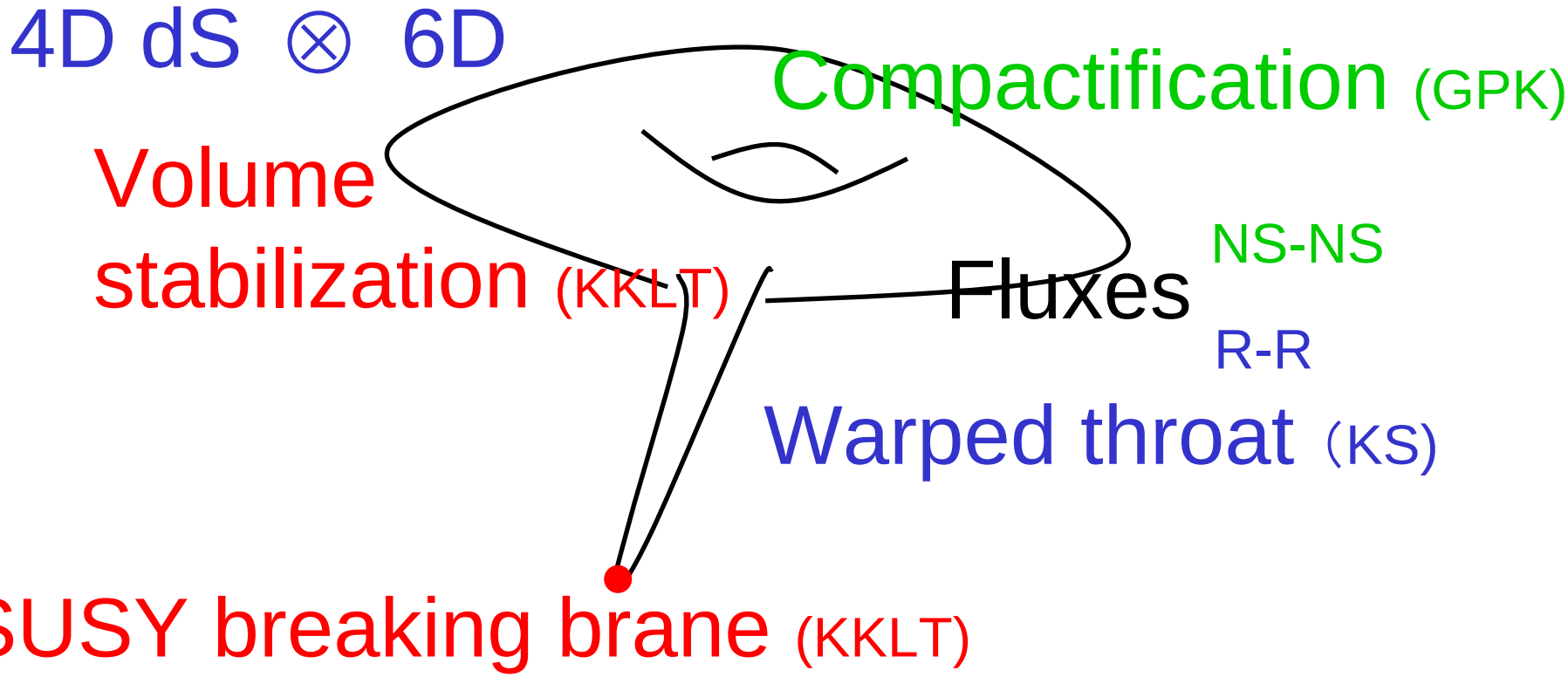
KKLT 4-dimensional de Sitter "solution"

- After stabilizing all moduli, anti-D-branes were introduced.
- Anti-D-branes or other SUSY breaking branes are indispensable!
- Without them, 4-dimensional cosmological constant would be negative and completely contradicts with cosmology.

SUSY breaking branes as Dark Matter

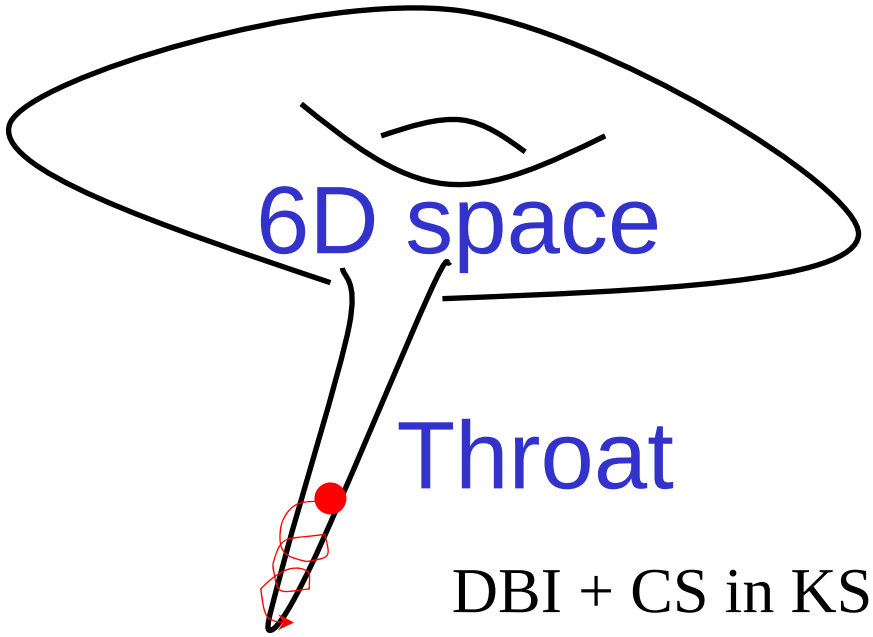
- **S.Mukohyama, "Anti-D-brane as Dark Matter in Warped String Compactification", Phys.Rev.D72, 061901 (2005) [hep-th/0505042].**

- What happens if SUSY breaking branes move in the extra 6 dimensions?



SUSY breaking branes as Dark Matter

- Falls toward the bottom of the throat, with rotation in the extra 5 dimensions.
- Behaves as DARK MATTER, from 4-dimensional viewpoint.



$\otimes$ 4D universe

DBI + CS in KS

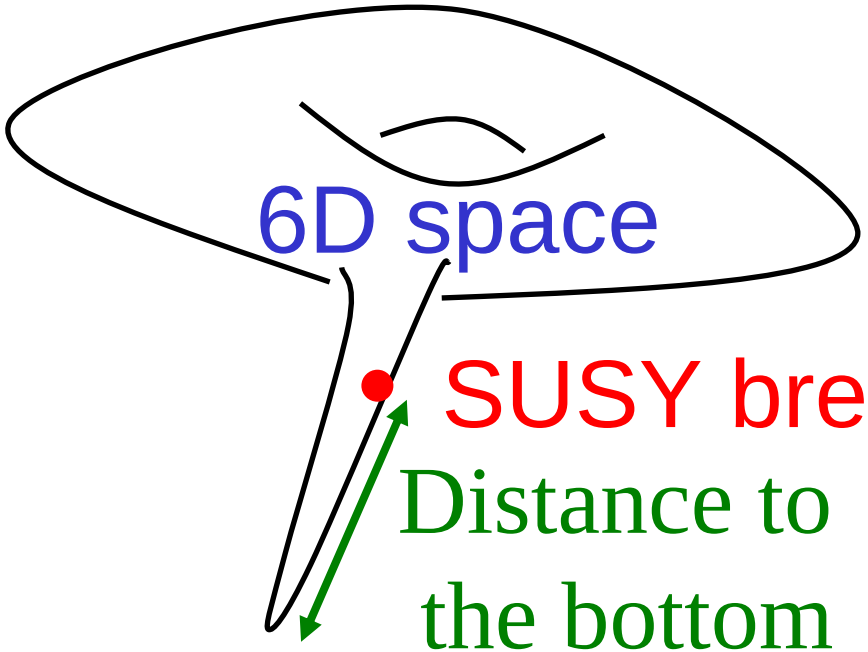
$\rho \propto a^{-3}$

brane motion = Dark Matter

Chaotic Inflation driven by brane motion

in progress, with L.Kofman

- Large motion of SUSY breaking brane
- In 4D, $V \sim \lambda\phi^4$



$\otimes$ 4D universe

SUSY breaking brane

Distance to the bottom = Inflaton

Anti-D3-brane action in KS geometry

DBI + CS

$$S_{\bar{D}3} = -T_3 \int d^4\xi e^{-\phi} \sqrt{-\det(G_{\alpha\beta} - B_{\alpha\beta})} - T_3 \int d^4\xi C_4$$

KS background + **Non-rel approximation**

$$\begin{aligned} S_{\bar{D}3} &= -T_3 \int d^4\xi \sqrt{-g^{(4)}} \left[\frac{\epsilon^{4/3}}{12K^2(\tau)} g^{(4)\alpha\beta} \partial_\alpha \tau \partial_\beta \tau + 2h^{-1}(\tau) \right] \\ &= -\int d^4\xi \sqrt{-g^{(4)}} \left[\frac{1}{2} g^{(4)\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi + V_{\bar{D}3}(\varphi) \right], \end{aligned}$$

$$\varphi \equiv \epsilon^{2/3} \sqrt{\frac{T_3}{6}} \int_0^\tau \frac{d\tau'}{K(\tau')}, \quad V_{\bar{D}3}(\varphi) \equiv \frac{2T_3}{h(\tau)}.$$

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3} \sinh\tau}$$

$$h(\tau) = 2^{2/3} \cdot (g_s M \alpha')^2 \epsilon^{-8/3} I(\tau)$$

$$I(\tau) = \int_\tau^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3}$$

Potential with non-rel. approx.

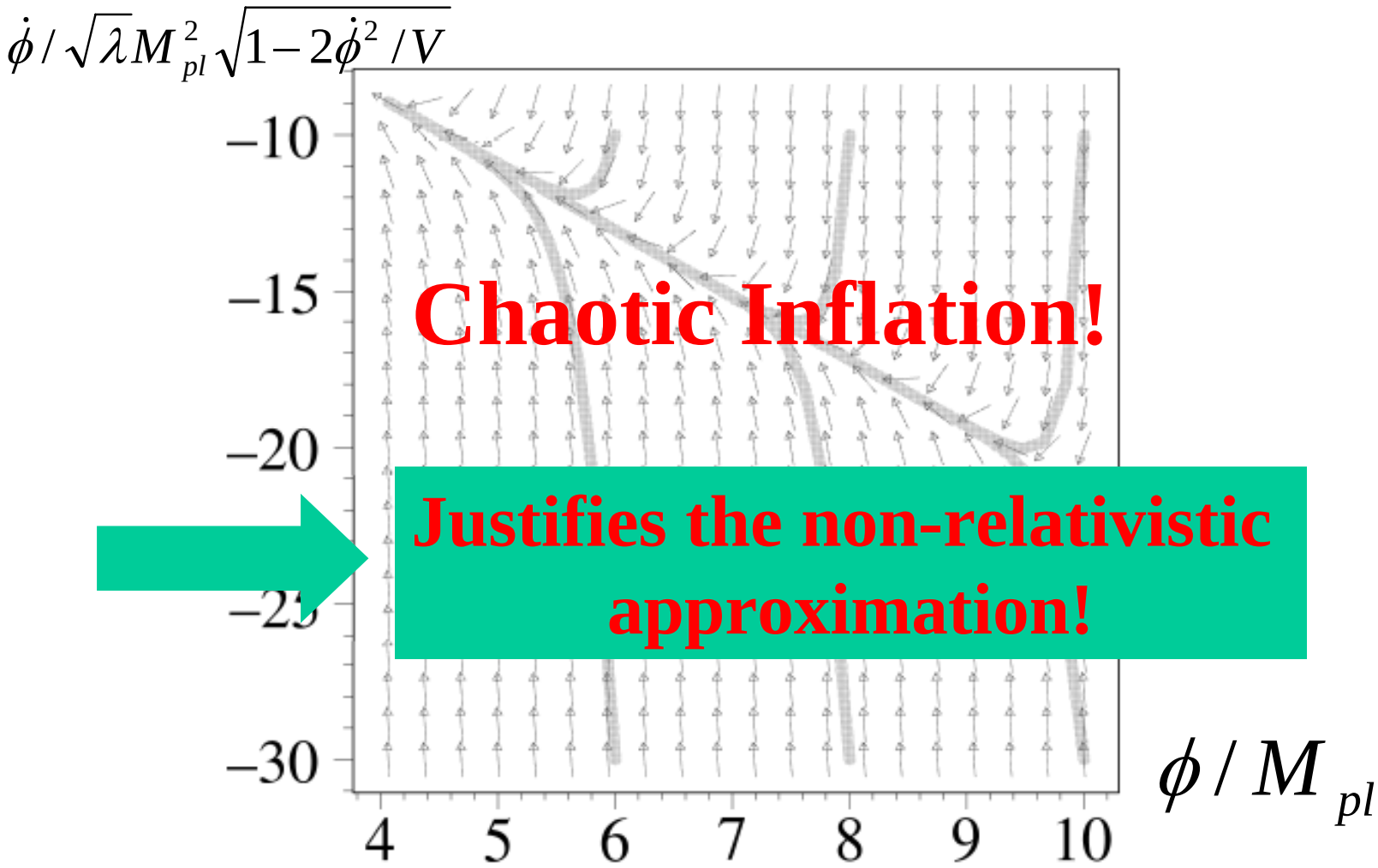
$$V(\varphi) \simeq \frac{(2/3)^4}{(g_s M\alpha')^2 T_3}\cdot\frac{\varphi^4}{\ln(\varphi/\varphi_0)} = \frac{\lambda_{\bar{D}3}\varphi^4}{1+C_{\bar{D}3}\ln(\varphi/M_{Pl})}$$

$$\varphi_0 = \frac{\epsilon^{2/3}\sqrt{3T_3}}{2^{5/6}} = \frac{\epsilon^{2/3}\sqrt{3}}{2^{5/6}(2\pi)^{3/2}\alpha' g_s^{1/2}} \simeq \frac{3^{1/2}I^{1/4}(0)}{2^{2/3}(2\pi)^{3/2}}\sqrt{\frac{M}{\alpha'}}\exp\left(-\frac{2\pi K}{3g_s M}\right),$$

$$\lambda_{\bar{D}3} = \frac{(2/3)^4(2\pi)^3 C_{\bar{D}3}}{g_s M^2} \simeq \frac{64\pi^2}{27MK}\left\{1+\frac{3g_s M}{4\pi K}\ln\left[\frac{2^{7/3}V_6}{3(2\pi)^4 I^{1/2}(0)g_s^2 M\alpha'^3}\right]\right\}^{-1},$$

$$C_{\bar{D}3} = \frac{1}{\ln(M_{Pl}/\varphi_0)} \simeq \frac{3g_s M}{2\pi K}\left\{1+\frac{3g_s M}{4\pi K}\ln\left[\frac{2^{7/3}V_6}{3(2\pi)^4 I^{1/2}(0)g_s^2 M\alpha'^3}\right]\right\}^{-1}.$$

Phase portrait for an anti-D3-brane without non-rel. approximation



Value of coupling constant λ ?

- CMB $\delta\rho/\rho \sim 10^{-5}$ requires $\lambda \sim 10^{-13}$
- For anti-D3-brane, $\lambda \sim 64\pi^2/27MK$, where M and K are values of fluxes (integers)... It seems difficult to make λ small enough... [c.f. $KM \sim \chi/24$. The known maximum value of χ = 1820448.]
- If we consider D7-brane wrapped around a 4-cycle, $\lambda \sim 32\pi g_s/27K^4[\ln(2\pi K/g_s M)+4\ln 2+2]$. Much better! [DBI & CS almost cancel, but the former (gravity) slightly wins because of the NS flux within the brane world-volume.]

Anti-D3 vs D7

- Different branes ~ different physics ~ various values of λ (coupling constant)
- Anti-D3-brane: RR charge is opposite to the background (KS geometry).
  Will be attracted towards the bottom of the throat.
- D7-brane: the sign of RR charge is the same as the background. But, gravity and RR field do not cancel exactly because of the wrapping along NS-NS flux.
  Gravity slightly wins and induces a small λ.

Examples of D7 potential

- D7-brane wrapped over a 4-cycle:
  $\lambda \sim 32\pi g_s/27K^4[\ln(2\pi K/g_s M)+4\ln 2+2]$
  λ can be as small as $\sim 10^{-13}$!
- Example 1:
  $g_s=0.2$, $M=25$, $K=3034$ ($\chi=1820400$)
  $\lambda \sim 1.1*10^{-13}$, $\phi_{max}^2/M_{Pl}^2 \sim 10^3$
- Example 2:
  $g_s=0.4$, $M=21$, $K=3612$ ($\chi=1820448$)
  $\lambda \sim 1.1*10^{-13}$, $\phi_{max}^2/M_{Pl}^2 \sim 2*10^3$

Open issues

- Effects of volume moduli stabilization
- Coupling to curvature
- Backreaction to the KS geometry
- WMAP data … assisted inflation?
- e.t.c.

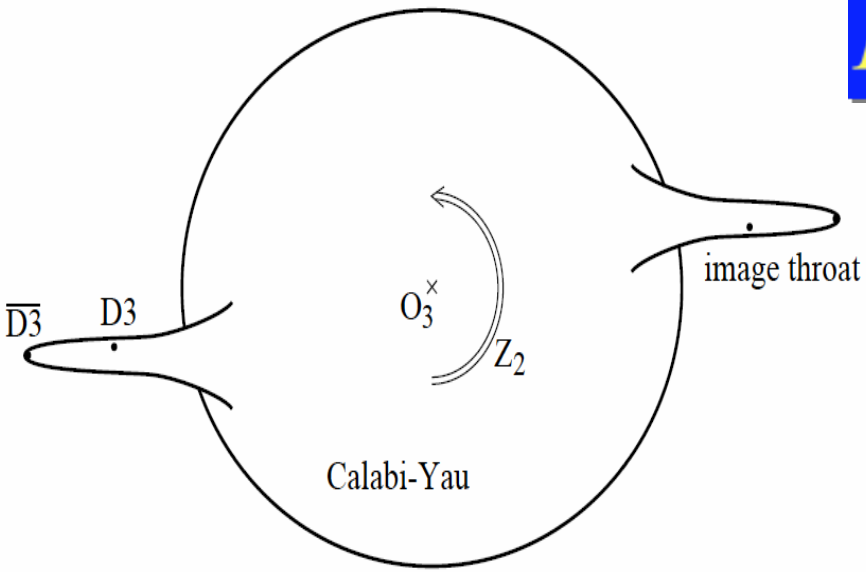
in progress, with L.Kofman

If successful, this would be the first realization of chaotic inflation in string theory!

Application II: “Fast-roll” inflation

The KKLMMT model

Kachru, Kallosh, Linde, Maldacena, McAllister, and Trivedi 2003



$$K = -3\log(\rho + \bar{\rho} - \phi\bar{\phi})$$

$$W = W_0 + e^{-a\rho}$$

$$m_\phi^2 = 2H^2$$

Meanwhile for inflation with a flat spectrum of perturbations one needs

$$m_\phi^2 \sim 10^{-2} H^2$$

This can be achieved by taking W depending on ϕ and by fine-tuning it at the level O(1%)

Avoiding the KKLMMT fine-tuning

in progress, with L.Kofman

- **$m_\phi^2 = 2H^2$ would stop inflation.**
- **This is due to the conformal coupling $-R\phi^2/12$.**
- However, people have not yet looked at **modification of Einstein equation.**
- We should take it into account!

Condition for inflation with conformal coupling

in progress, with L.Kofman

- **Usual slow roll condition**

$$\epsilon \ll 1, \quad |\eta| \ll 1$$

$$\epsilon \equiv \frac{1}{2\kappa^2}\left(\frac{V'}{V}\right)^2 \quad \eta \equiv \frac{V''}{\kappa^2 V}$$

+ an additional condition

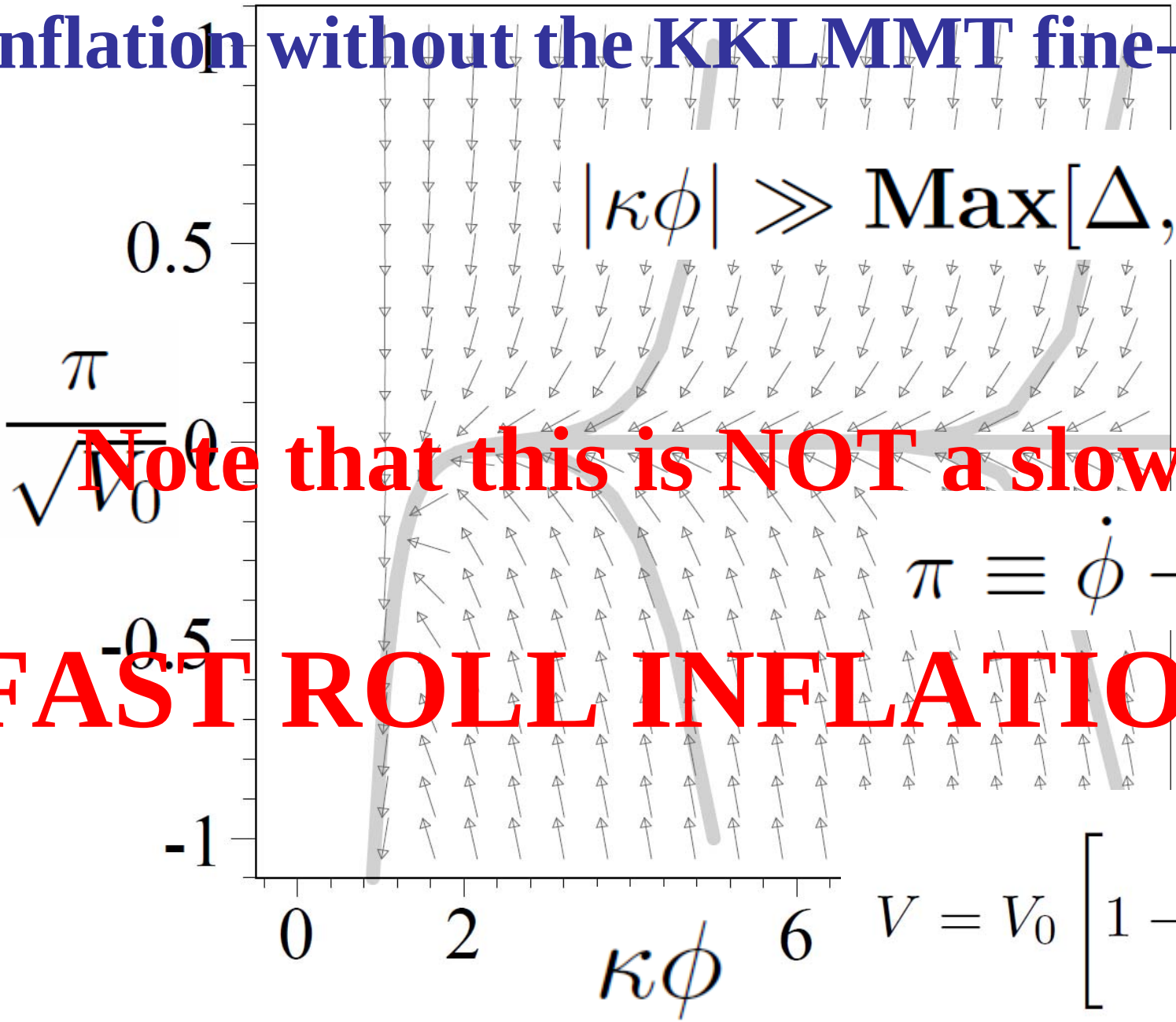
$$|\tilde{\epsilon}| \ll 1$$

$$\tilde{\epsilon} \equiv \frac{V'\phi}{2V}$$

- The latter condition is not satisfied by power-law potentials.
- **The D-antiD potential satisfies it!**

in progress, with L.Kofman

Inflation without the KKLMMT fine-tuning!



$$|\kappa\phi| \gg \mathrm{Max}[\Delta, \Delta^{2/3}]$$

Note that this is NOT a slow roll!

$$\pi \equiv \dot{\phi} + H\phi$$

FAST ROLL INFLATION!!!

$$V = V_0 \left[1 - \left(\frac{\Delta}{\kappa\phi}\right)^4\right]$$

e-foldings & mass hierarchy

- e-foldings:

  a ϕ ~ const.

  $N \sim \ln (\phi_i / \phi_e)$

- Mass hierarchy a la Randall-Sundrum :

  $M / M_{pl} \sim e^{-N}$

- **Enough inflation vs TeV gravity:**

  $N \sim 62 - \ln (M / 10^{16}\text{GeV})$

  $M \sim$ TeV

  These conditions are **equivalent!**

Density perturbation

- $\delta\phi$ does not generate scale-invariant density perturbation.
- **Angular position of the D3 in S^3** is a light field, and thus its spectrum is scale-invariant.
- This generates long-wavelength spatial variation of the end-of-inflation time hypersurface.
- **This "modulated reheating" generates scale-invariant density perturbation!**

Hybrid inflation seems possible without the KKLMMT fine-tuning!

Kofman and Mukohyama, to appear

Application III: Higgs phase of gravity

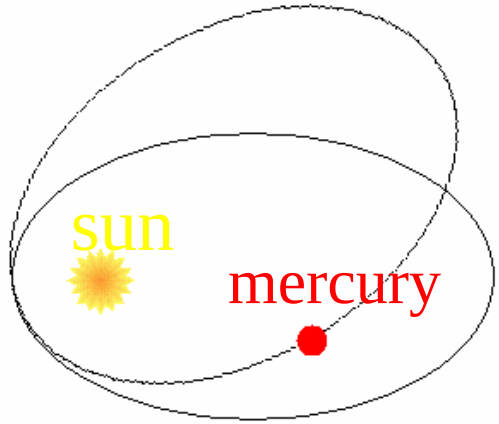
Motivation

- Gravity at long distances
  Flattening galaxy rotation curves
  Dimming supernovae
  accelerating universe

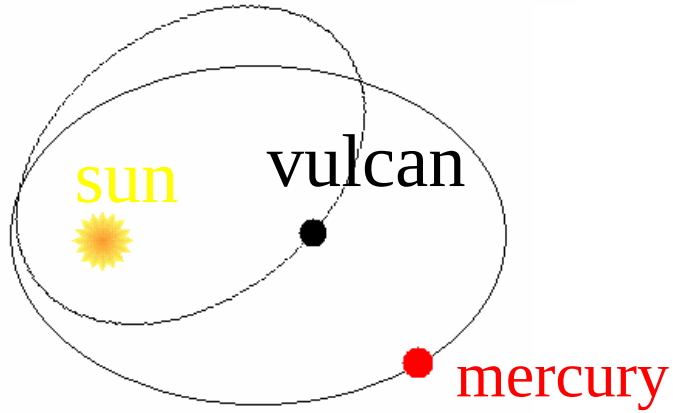
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Historical remark:

Precession of perihelion observed in 1800's…



which people tried to explain with a "dark planet", Vulcan,



But the right answer wasn't "dark planet", it was "change gravity" from Newton to GR.

Can we change gravity in IR instead of introducing DE/DM?

- Change theory?

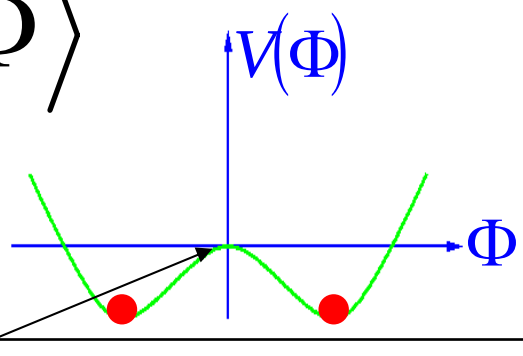
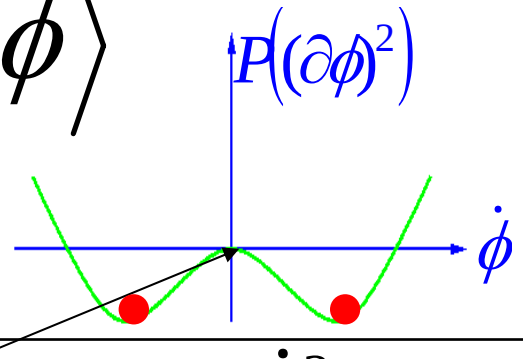
  Macroscopic UV scale…

- Change state (phase)?

  Higgs phase of gravity

  The simplest: ghost condensation

  Arkani-Hamed, Cheng, Luty and Mukohyama, hep-th/0312099

	Higgs Mechanism	***Ghost Condensation*** Arkani-Hamed, Cheng, Luty and Mukohyama, hep-th/0312099
Order Parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-m^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V'=0, V''>0	P'=0, P''>0
Spontaneous breaking	Gauge symmetry	Lorents symmetry (Time translation)
Modifying	Gauge force	Gravitational force
New potential	Yukawa-type	Oscillating in space Growing in time

Systematic construction of Low-energy effective theory

Backgrounds characterized by

- $\left\langle \partial_\mu \phi \right\rangle \neq 0$ and timelike

- Background metric is maximally symmetric, either Minkowski or dS.

Gauge choice: $\phi(t,\vec{x}) = t. \quad \pi \equiv \delta\phi = 0$

(Unitary gauge)

Residual symmetry: $\vec{x} \to \vec{x}'(t,\vec{x})$

⟹ Write down most general action invariant under this residual symmetry.

(⟹ Action for π: undo unitary gauge!)

Start with flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Under residual ξ^i

$$\delta h_{00} = 0, \delta h_{0i} = \partial_0 \xi_i, \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

Action invariant under ξ^i

$$\begin{cases} (h_{00})^2 & \text{OK} \\ \cancel{(h_{0i})^2} & \\ K^2, K^{ij}K_{ij} & \text{OK} \end{cases}$$

Beginning at quadratic order, since we are assuming flat space is good background.

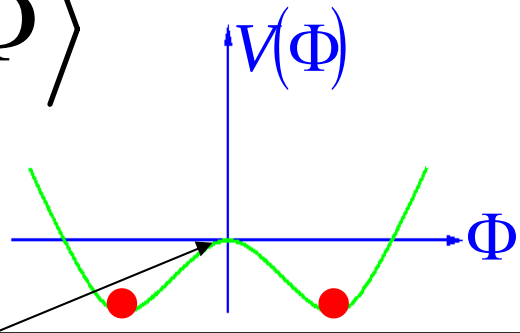
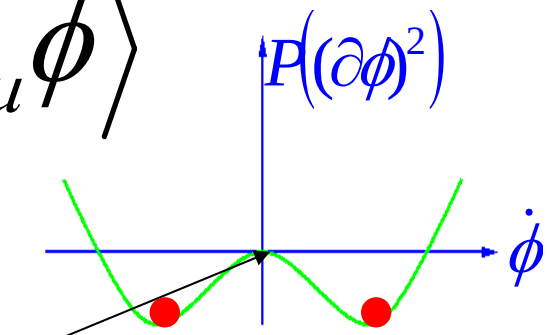
$$K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}\right)$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4\left\{(h_{00})^2 - \frac{\alpha_1}{M^2}K^2 - \frac{\alpha_2}{M^2}K^{ij}K_{ij} + \cdots\right\}$$

Action for π

$$\xi^0 = \pi \qquad \begin{cases} h_{00} \to h_{00} - 2\partial_0\pi \\ K_{ij} \to K_{ij} + \partial_i\partial_j\pi \end{cases}$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4\left\{(h_{00} - 2\dot{\pi})^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2\pi\right)^2 - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i\vec{\nabla}^j\pi\right)\left(K_{ij} + \vec{\nabla}_i\vec{\nabla}_j\pi\right) + \cdots\right\}$$

	Higgs Mechanism	***Ghost Condensation*** Arkani-Hamed, Cheng, Luty and Mukohyama, hep-th/0312099
Order Parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
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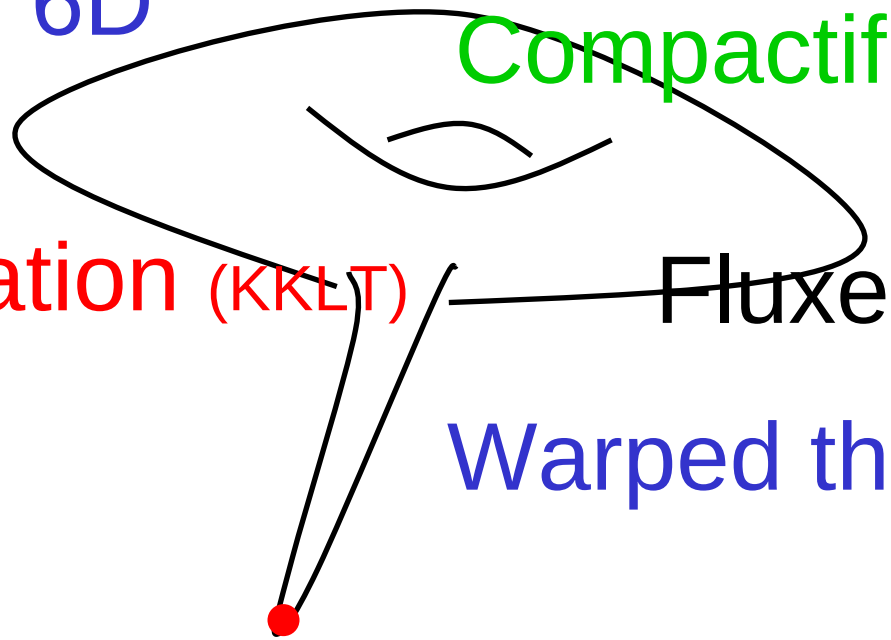
Modifying	Gauge force	Gravitational force
New potential	Yukawa-type	Oscillating in space Growing in time

KKLT setup

4D dS ⊗ 6D

Compactification (GPK)

Volume stabilization (KKLT)



Fluxes

NS-NS

R-R

Warped throat (KS)

Anti-D3-branes (KKLT)

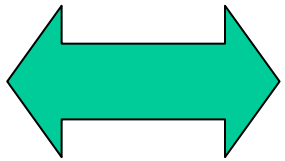
➡ **Non-SUSY NS5-brane**

Kachru, Pearson & Verlinde (2002)

Correspondence principle

Horowitz & Polchinski (1997)

Stringy Object

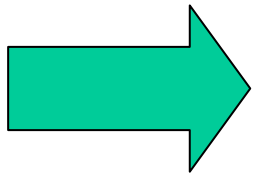


Black-Brane

Size > R_{grav}

Size < R_{grav}

Non-SUSY NS5-brane



Black-Brane

$$\left(M_{RR}/\bar{N}_3\right)^2 \gtrsim g_s \bar{N}_3 \gg 1$$

Mukohyama, hep-th/0610254

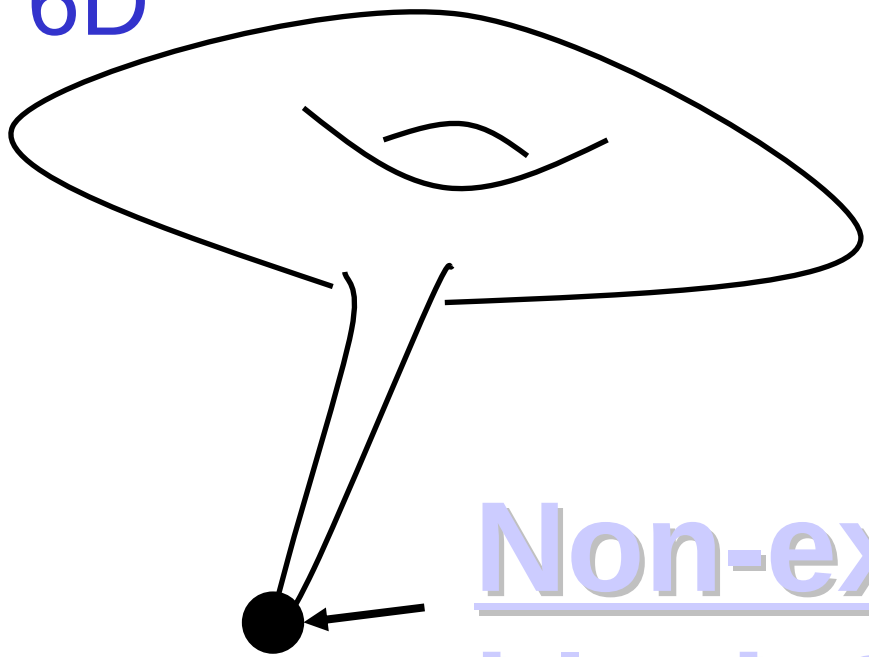
M_{RR}: # of R-R flux

$\bar{N}_3$: # of $\overline{D3}$'s

g_s : string coupling

Black brane at the tip

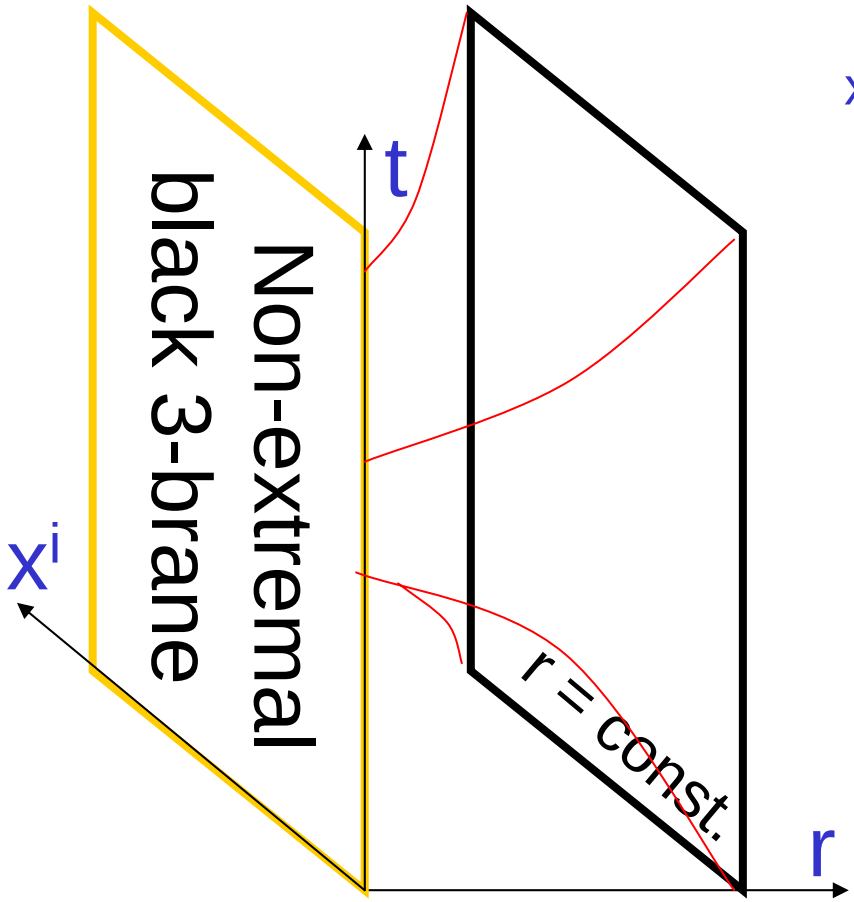
4D dS $\otimes$ 6D

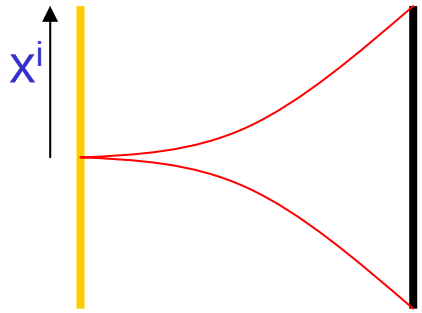


Non-extremal black 3-brane

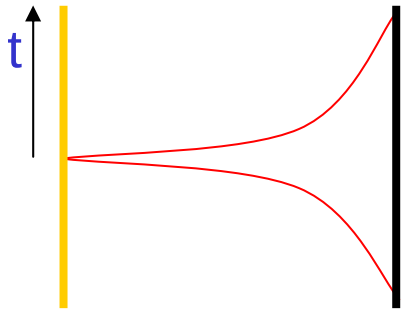
Spontaneous Lorentz breaking

- The (3+1)-dim spacetime is spanned by (t , x^i).





Projection onto
t = const. surfaces



Projection onto
x^i = const. surfaces

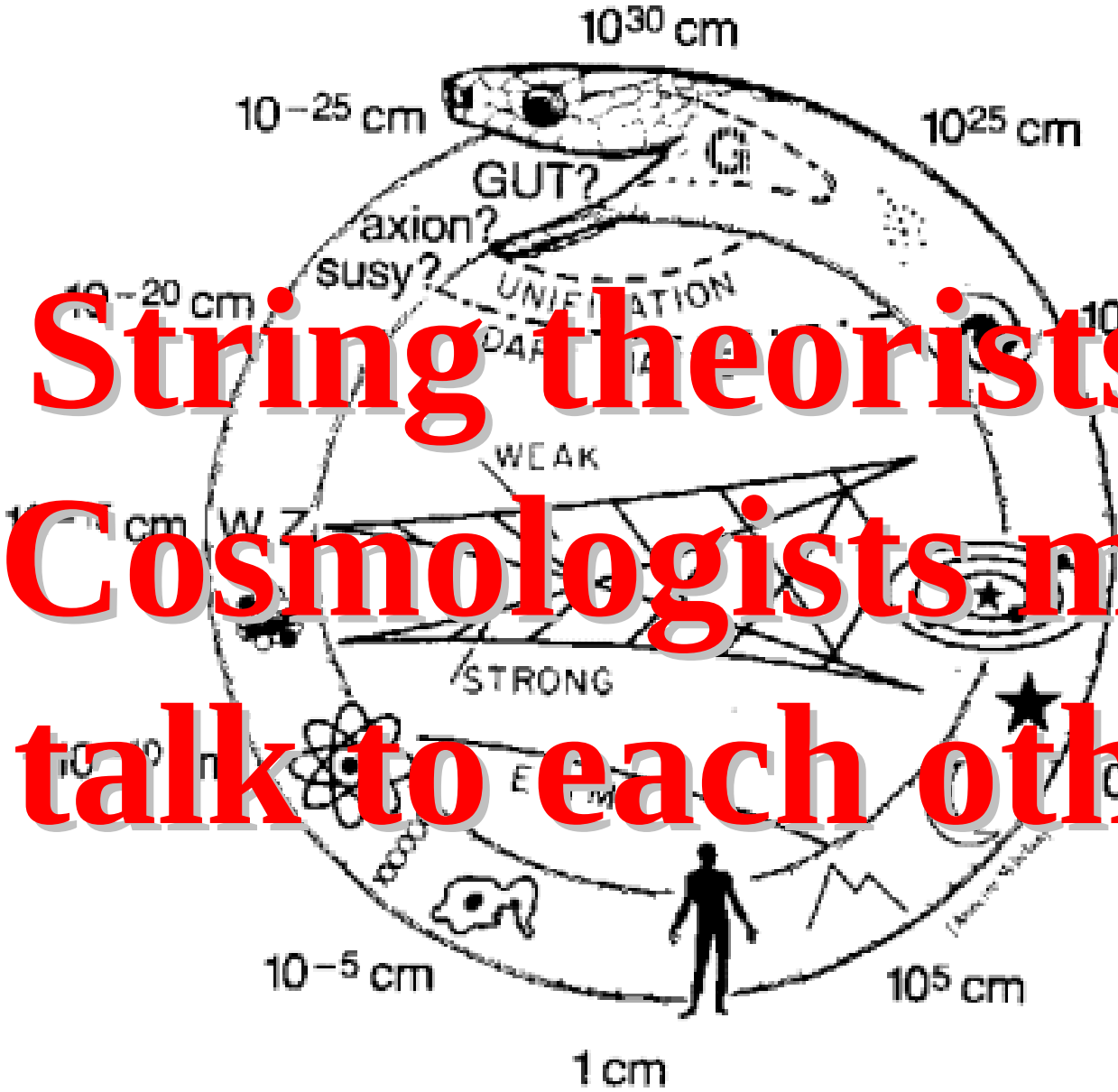
Warp factors for the tt-component and the ij-components are different.

Spontaneous Lorentz breaking!

Gauged Ghost Condensation

Summary

- It seems that we can really enjoy **cosmology in the framework of string theory.**
- Application I: In the KKLT setup, **the motion of SUSY breaking branes** can be an inflaton. This would be **the first realization of chaotic inflation** in string theory.
- Application II: **“Fast-roll” inflation of hybrid type** is possible **without the KKLLMT fine-tuning. The mass hierarchy and e-foldings are related!**
- Application III: **Higgs phase of gravity** may be realized.
- A lot of interesting subjects are still remaining!



String theorists & Cosmologists must talk to each other!

The Cosmic Uroboros by Sheldon Glashow