Mesons as Open Strings

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1 Introduction

mesons ($N_f = 2$, Isovector)

parity

charge conjugation

spin

J^{PC}

mass (MeV)

$\triangle$ ... not established

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	985	1474				
$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				
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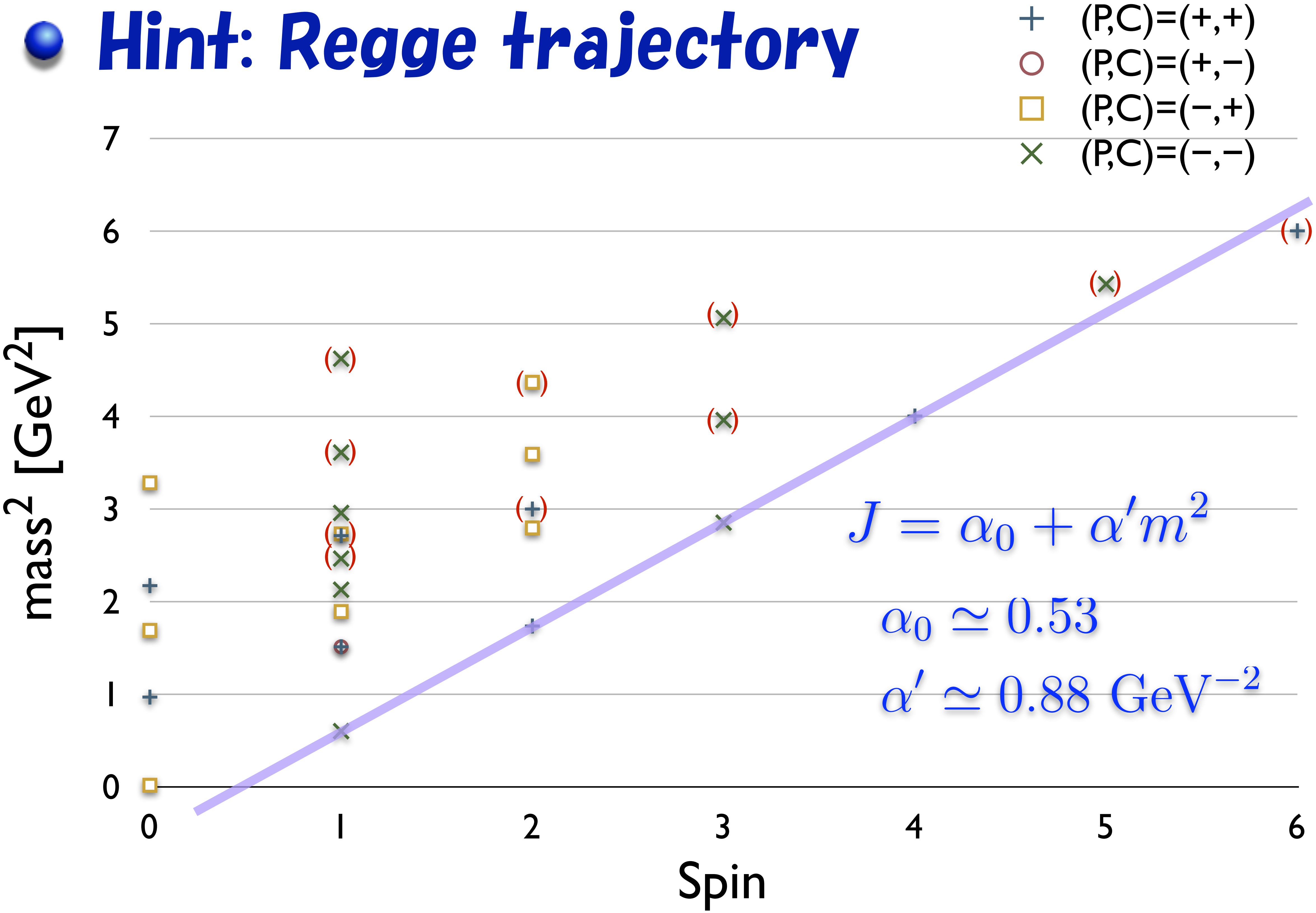
$3^{--}(\rho_3)$	1689	$1990^{\triangle}$	$2250^{\triangle}$			
$4^{++}(a_4)$	2001					
$5^{--}(\rho_5)$	$2330^{\triangle}$					
$6^{++}(a_6)$	$2450^{\triangle}$					

Q: How can we understand these numbers?

Hint: Regge trajectory



Mesons are strings !?

Difficulties in the old days

- Consistent in 10 dim space-time
- ∃ massless particles with J = 1 and J = 2
  (open) (closed)

$$J = \alpha_0 + \alpha' m^2 \qquad \alpha_0 = 1\ ! \quad \text{(for open string)}$$

Not consistent with meson spectrum !?

Gauge / String duality suggests

- 4 dim QCD $\Longleftrightarrow_{\text{dual}}$ 10 dim string theory
  (in a certain curved background)
- massive mesons in 4 dim $\Longleftrightarrow_{\text{dual}}$ massless gauge field in 10 dim

The above difficulties can be solved !!

Holographic QCD

- 4 dim QCD $\Longleftrightarrow$ 10 dim string theory
  dual (in a certain curved background)
  holographic QCD

D4/D8-branes in type IIA string theory

(reviewed in the next section) [T.Sakai and S.S. 04]

- mesons are open strings on D8
- $\pi, \ \rho, \ a_1,$ etc. are obtained from the massless mode

0^{-+} 1^{--} 1^{++}

$$m_{a_1}/m_\rho \simeq \begin{cases} 1.53 & (\text{theory}) \\ 1.59 & (\text{exp}) \end{cases}$$

Q: What about the other mesons ?

→ Consider massive modes (excited strings)

Plan

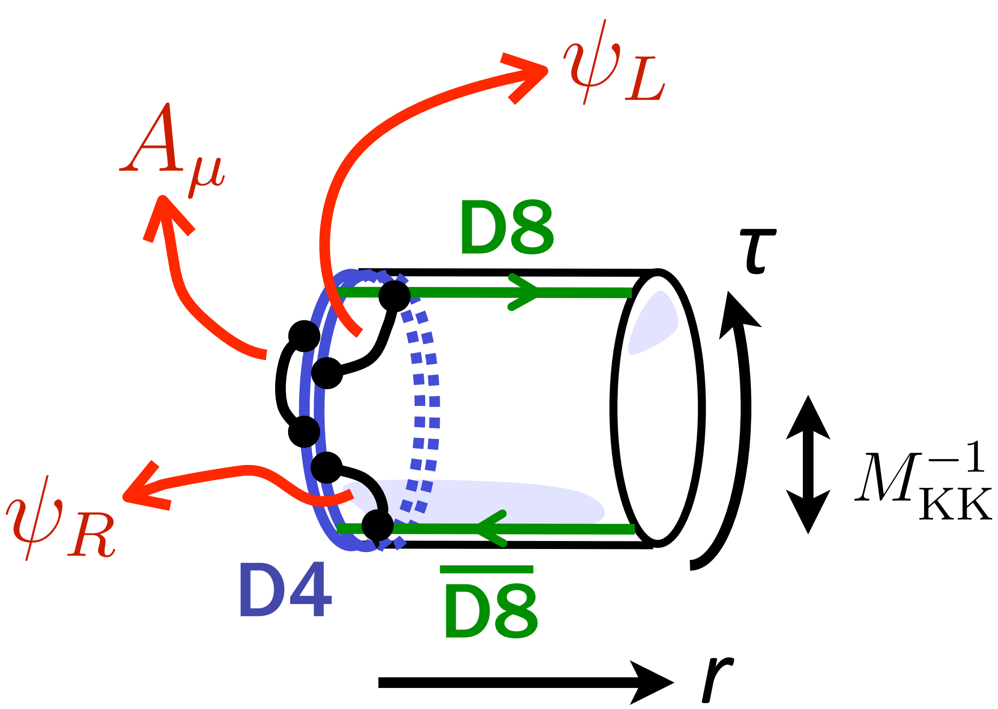
1. Introduction
2. Brief review of the model
3. Meson spectrum
4. Comparison with data
5. Discussion

2 Brief review of the model [T. Sakai and S.S. 04]

Brane configuration

	x^0	x^1	x^2	x^3	τ	x^5	x^6	x^7	x^8	x^9
D4 x N_C	○	○	○	○	○					
D8-$\overline{\text{D8}}$ x N_f	○	○	○	○		○	○	○	○	○

S^1 with ~~SUSY~~ b.c.

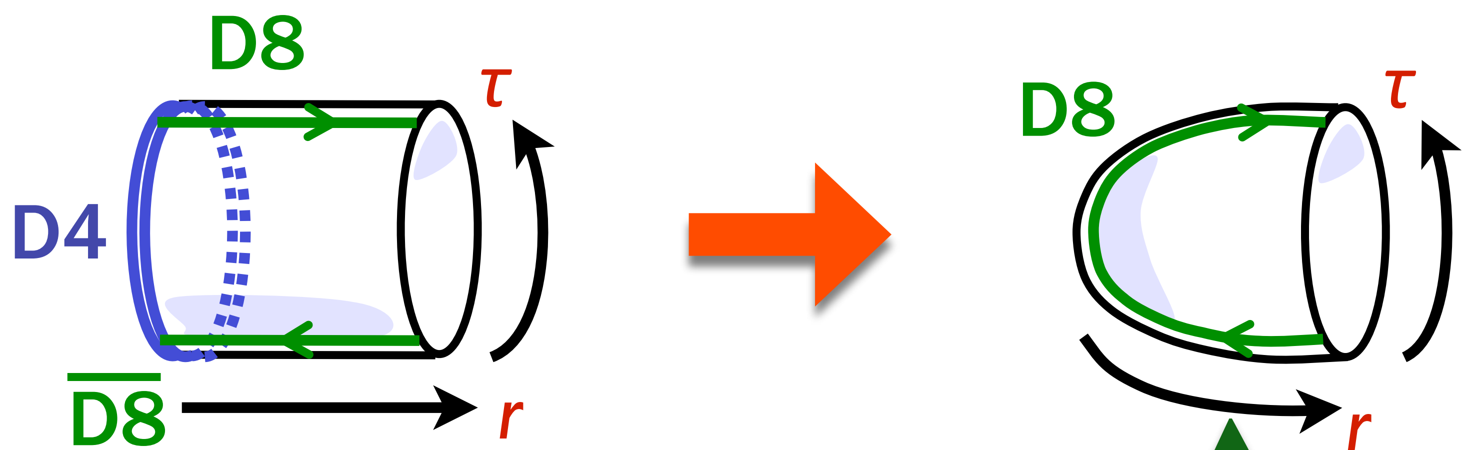


(radial direction of $x^{5\sim 9}$)

4 dim $U(N_C)$ QCD
with N_f massless quarks
(at low energy)

Holographic description

- replace D4 with the corresponding curved background
  [Witten 98]
- D8 are treated as probe brane (assuming $N_c \gg N_f$)

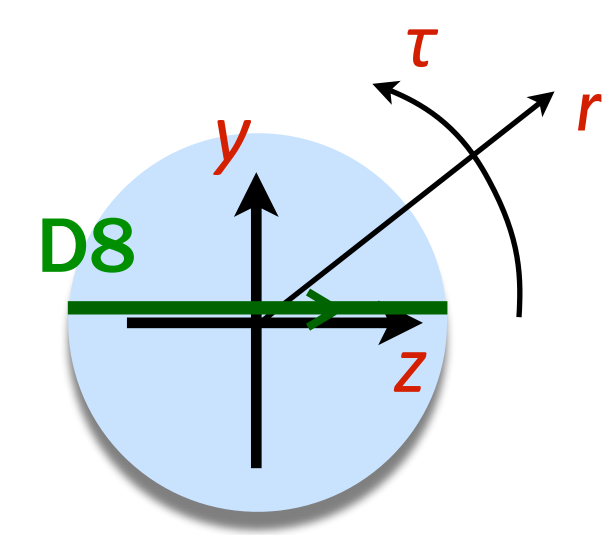


$$\underset{x^{0\sim3}}{\mathbf{R}^{1,3}} \times \underset{\tau}{S^1} \times \underset{(r,\,\theta^{1\sim4})}{\mathbf{R}^5}$$

S^4

$$\underset{x^{0\sim3}}{\mathbf{R}^{1,3}} \times \underset{(r,\tau)}{\mathbf{R}^2} \times \underset{\theta^{1\sim4}}{S^4} \quad \text{(topology)}$$

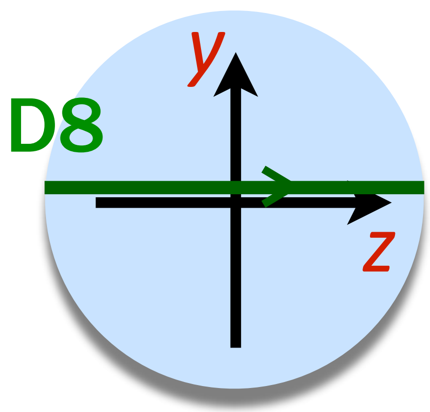
$$\begin{cases} y = r \sin \tau \\ z = r \cos \tau \end{cases}$$



Hadrons in the model

$$\mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$$

$x^{0\sim 3}$ (z,y)



D8 are extended along $\underbrace{(x^\mu, z)}^{\text{5dim}}$ x S^4

(μ=0～3)

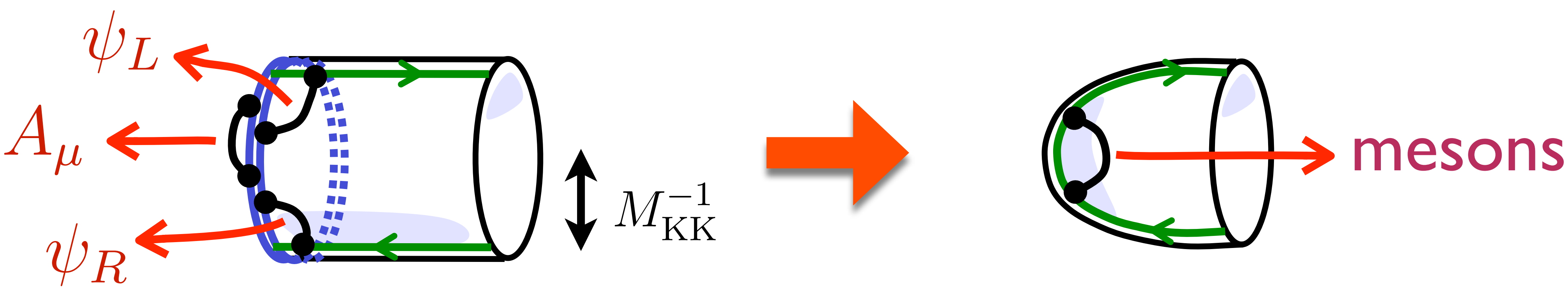
- closed strings ⟷ glueballs
- open strings on D8 ⟷ mesons
- D4 wrapped on S^4 ⟷ baryons

today's topic

QCD mesons vs artifacts

- Our brane config. is invariant under *SO(5)* ↷ S^4
- quarks and gluons are invariant under *SO(5)*

  (non-invariant states are massive KK-modes)

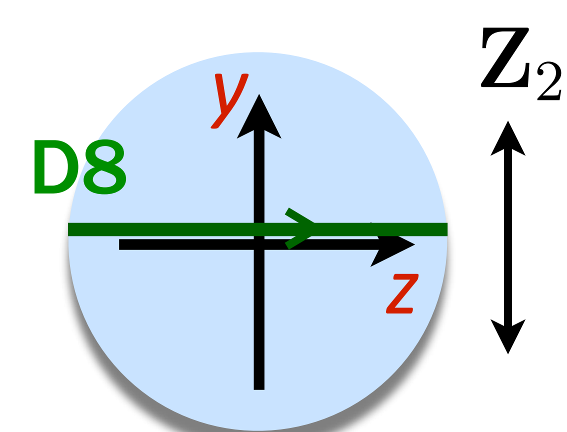


- QCD mesons are *SO(5)* invariant states

  (non-invariant states are KK-artifacts)

- Similarly, we can show that QCD mesons are invariant under Z_2 sym generated by $I_{y9}(-1)^{F_L}$

  I_{y9}: $(y, x^9) \longrightarrow (-y, -x^9)$ $\quad$ $(\tau \longrightarrow -\tau)$



→ Consider $SO(5) \rtimes \mathbf{Z}_2$ invariant states

3 Meson spectrum

Consider open strings attached on D8

- **Massless mode** (light-cone gauge: $x^{\pm} = x^0 \pm x^1$)
  - $\psi^{I}_{-1/2}|0\rangle_{\rm NS}$ $(I = 2, 3, z)$ → 5 dim gauge field $A_\mu,\ A_z$
  - $\psi^{A}_{-1/2}|0\rangle_{\rm NS}$ $(A = y, \underbrace{6, 7, 8, 9}_{S^4})$
  - $|\alpha\rangle_{\rm R}$ spinor

  → not invariant under $SO(5) \rtimes \mathbf{Z}_2$

- **KK decomposition along *z* direction**

$$A_\mu(x^\mu, z) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x^\mu)\psi_n(z)$$

$$A_z(x^\mu, z) = \sum_{n=0}^{\infty} \varphi^{(n)}(x^\mu)\phi_n(z)$$

complet sets

	$B_\mu{}^{(1)}$	$B_\mu{}^{(2)}$	$B_\mu{}^{(3)}$	...	$\varphi^{(0)}$	$\varphi^{(1)}$	...
J^{PC}	1^{--}	1^{++}	1^{--}	...	0^{-+}	eaten	
	ρ	a_1	ρ'	...	π		

[T.Sakai and S.S. 04]

First excited massive modes

First, we consider flat space-time limit

(justified when ’t Hooft coupling is very large)

$SO(5) \rtimes \mathbf{Z}_2$ invariant states:

$(I = 2, 3, z)$

- $\psi^I_{-3/2}|0\rangle_{\rm NS}$
- $\alpha^{(I}_{-1}\psi^{J)}_{-1/2}|0\rangle_{\rm NS}$
- $\alpha^{[I}_{-1}\psi^{J]}_{-1/2}|0\rangle_{\rm NS}$
- $\psi^I_{-1/2}\psi^J_{-1/2}\psi^K_{-1/2}|0\rangle_{\rm NS}$
- $\alpha^y_{-1}\psi^y_{-1/2}|0\rangle_{\rm NS}$
- $\displaystyle\sum_{\underbrace{a=6,7,8,9}_{S^4}} \alpha^a_{-1}\psi^a_{-1/2}|0\rangle_{\rm NS}$

5dim field	$SO(4)$ little gr
h_{MN} $(M, N = 1, 2, 3, z)$	$\square\!\square$
A_{MNP}	$\begin{array}{c}\square\\\square\\\square\end{array}$
$\varphi^{[1]}$	1
$\varphi^{[2]}$	1

KK decomposition along z direction

$$h_{MN}(x^\mu, z) = \sum_{n=0}^{\infty} h_{MN}^{(n)}(x^\mu)\phi_n(z) \qquad \text{etc.}$$

lowest modes: (i , j , k = 1, 2, 3)

	$h_{ij}^{(0)}$	$h_{iz}^{(0)}$	$h_{zz}^{(0)}$	$A_{ijk}^{(0)}$	$A_{ijz}^{(0)}$	$\varphi^{[1,2](0)}$
J^{PC}	2^{++}	1^{+-}	0^{++}	0^{-+}	1^{--}	0^{++} x 2

Second excited massive mode

lowest modes:

J^{PC}	3^{--}	2^{++}	2^{--}	2^{-+} x 2	1^{--} x 7	1^{++} x 3	1^{+-} x 4	1^{-+}	0^{++} x 2	0^{-+} x 6

Mass formula (naive shortcut)

- Flat space-time limit:

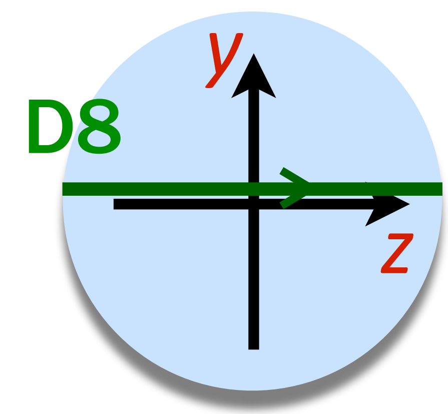
$$\boxed{m_0^2 = \frac{N}{\alpha'}}$$

$$\left(\alpha'^{-1} = \frac{4}{27}\lambda M_{\mathrm{KK}}^2\right)$$

$N = 0, 1, 2, \cdots$: excitation level

- Massive particle in curved space-time

$$S = -m_0 \int \sqrt{g_{tt}}\, dt = -m_0 (1+z^2)^{1/4} \int dt$$



→ particle in potential: $V(z) = m_0(1+z^2)^{1/4}$

→ $$\boxed{M_n \simeq m_0 + \frac{1}{\sqrt{2}}\left(n + \frac{1}{2}\right) M_{\mathrm{KK}} + \mathcal{O}(\lambda^{-1/2})}$$ $n = 0, 1, 2, \cdots$

harmonic oscillator approx.

- More careful analysis shows that the *O*(1) term is not affected by the RR-flux, α' correction, etc.

4 Comparison with data

Massless mode

[T.Sakai and S.S. 04]

	$B_\mu^{(1)}$	$B_\mu^{(2)}$	$B_\mu^{(3)}$	$B_\mu^{(3)}$	...	$\varphi^{(0)}$
J^{PC}	1^{--}	1^{++}	1^{--}	1^{++}	...	0^{-+}
	ρ	a_1	ρ'	a'_1	...	π
mass (MeV)	[776]	1189	1607	2024	...	0

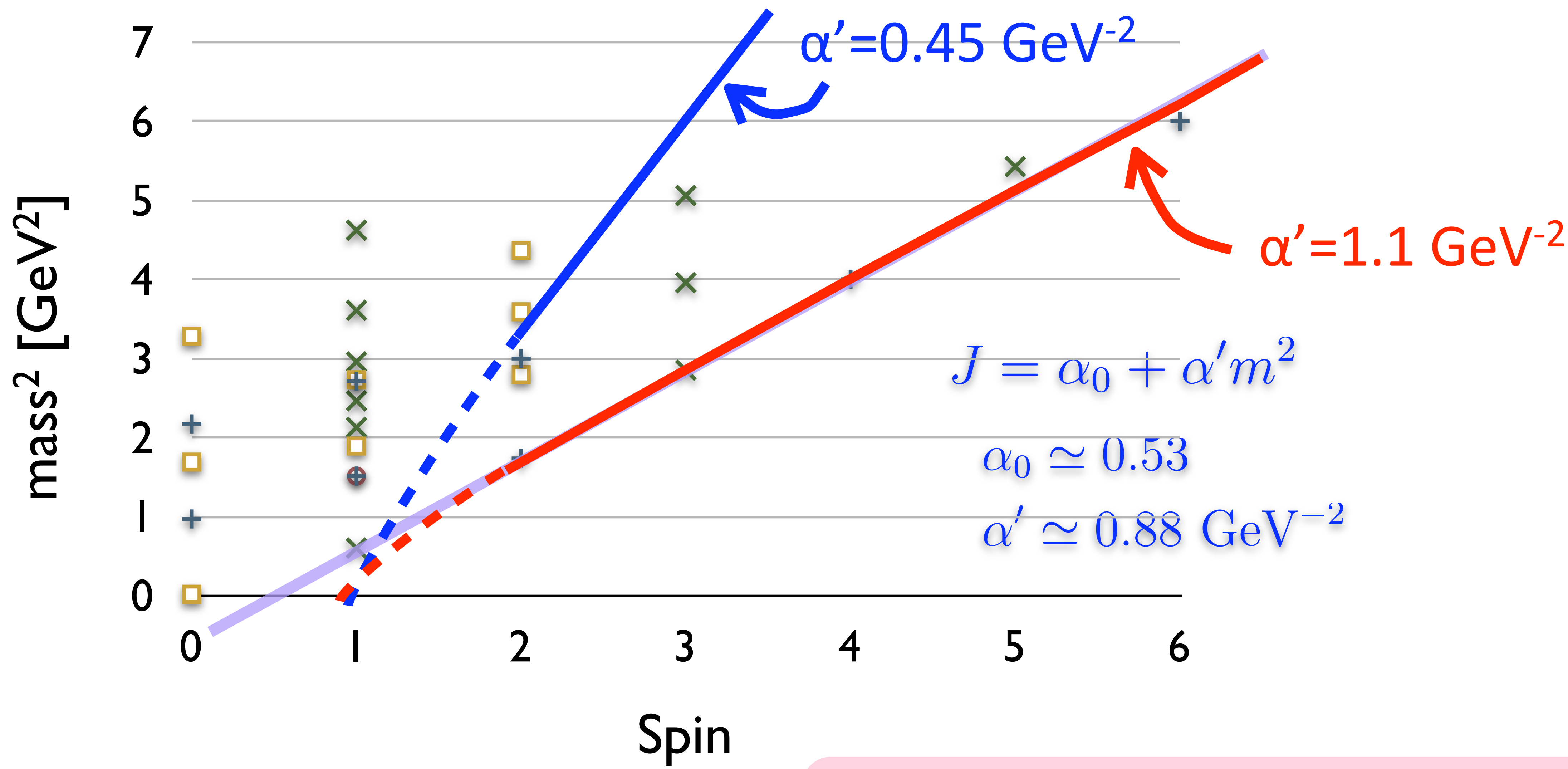
good!

?

experiment:

$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
$1^{++}(a_1)$	1230	$1647^{\triangle}$				

Regge trajectory



$$M_n \simeq \sqrt{\frac{N}{\alpha'} + \frac{1}{\sqrt{2}}\left(n+\frac{1}{2}\right)} M_{\mathrm{KK}} \quad \xrightarrow{N = J-1,\ n=0} \quad \boxed{J \simeq 1 + \alpha' M^2 - \frac{\alpha'}{\sqrt{2}} M_{\mathrm{KK}} M + \mathcal{O}(\lambda^{-1})}$$

- If we use f_π to fit λ, we obtain α' = 0.45 GeV^{-2}. This is unfortunately too small.
- If we set α'=1.1 GeV^{-2} we get very good fit.

First excited states (*N*=1, *n*=0)

J^{PC}	2^{++}	1^{+-}	1^{--}	0^{-+}	0^{++} x 3

?

2^{++}, 1^{+-}, 0^{-+}, 0^{++} cannot be *N*=0

$\Rightarrow$ good candidates for N=1

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	*~~985~~	1474				
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: *N*=0

: *N*=1

- degenerate around 1300 MeV
- *a_0(980) is considered to be a four quark state.

Second excited states (*N*=2, *n*=0)

J^{PC}	3^{--}	2^{++}	2^{--}	2^{-+} x 2	1^{--} x 7	1^{++} x 3	1^{+-} x 4	1^{-+}	0^{++} x 2	0^{-+} x 6

? ★ ? ? ★ ? ★ ?

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: *N*=0

: *N*=1

: *N*=2

- ★: prediction ?
- degenerate around 1700 MeV
- * π_1 (1400) is claimed to be a four quark state. (could be hybrid)

Summary

$0^{-+}(\pi)$	135	1300	1812			
$0^{++}(a_0)$	~~985~~	1474				
$1^{--}(\rho)$	776	1459	$1570^{\triangle}$	1720	$1900^{\triangle}$	$2150^{\triangle}$
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: N=0

: N=1

: N=2

I think this is non-trivial.
What do you think?

5 Discussion

- Mesons are Strings
- Wikipedia says:

Problems and controversy

Although string theory comes from physics, some say that string theory's current untestable status means that it should be classified as more of a mathematical framework for building models as opposed to a physical theory.

..... Yet, for all this activity, not a single new testable prediction has been made, not a single theoretical puzzle has been solved. ....

Don't criticize string theory
in this way anymore !