

Branes and Fluxes in Cascading Gauge Theories & SUSY breaking

Workshop @ Rikkyo Univ

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Based on works with Aki Hashimoto, Peter Ouyang, and Ofer Aharony

Introduction

- The Gauge/Gravity Duality

Gauge Theory in D -dimensions = Gravitational Theory in $(D+1)$ -dimensions

(\leftarrow) A geometric reformulation of Quantum Mechanics/Field Theory

(\rightarrow) A holographic formulation of Quantum Gravity (finite N)

- In String Theory, realized as the open/closed string duality in the near-horizon brane spacetime

- Typically, two parameters (i) N (rank of gauge group = # of branes)

- (ii) λ ('t Hooft coupling \simeq string tension)

- \rightarrow In the large N and λ limit (well studied),

Strong Coupling Gauge Theory = Classical Gravity in Weakly Curved Spacetime

- (Best understood) canonical example of Gauge/Gravity duality; AdS_5/CFT_4

$$\text{String Theory on } AdS_5 \times S^5 = 4d \mathcal{N} = 4 SU(N) \text{ SYM}$$

- Today's focus

Based on developments in AdS_4/CFT_3 in the last couple of years

– following BLG + ABJM's discovery of the local Lagrangian description of 3d SCFT (which took a decade)

- Study gravity duals of 3d SYM + CS theory

– to understand RG flow to (non-)conformal theories, Seiberg-like (strong-weak coupling) dualities, and dynamical SUSY breaking (DSB)

(An interesting example where one can study nontrivial gauge dynamics via gravity duals)

AdS_4/CFT_3 (BLG + ABJM)

$$U(N)_k \times U(N)_{-k} \text{ Chern-Simons-Matter} = \text{M-theory on } AdS_4 \times S^7/Z_k$$

valid when $k \ll N^{1/5}$ and large N

$$U(N)_k \times U(N)_{-k} \text{ Chern-Simons-Matter} = \text{Type IIA on } AdS_4 \times CP^3$$

for larger values of k , $N^{1/5} \ll k \ll N$ and large N

Note: $\mathcal{N} = 8$ for $k = 1, 2$ and $\mathcal{N} = 6$ for $k > 2$

- The type IIA reduction:

$$ds_{IIA(M)}^2 = \frac{R^2}{4} ds_{AdS_4}^2 + R^2 \left(ds_{CP^3}^2 + \frac{1}{k^2} (d\varphi + k\omega)^2 \right)$$

where $ds_{S^7/Z_k}^2 = \frac{1}{k^2} (d\varphi + k\omega)^2 + ds_{CP^3}^2$ ($S^7/Z_k = U(1)$ fibration over CP^3)

- The CSM theory 't Hooft coupling $\lambda = N/k$ is *strong* in the type IIA and M-theory descriptions
- Schematic form of Lagrangian:

$$S = \int \text{Tr} \left[k \left(A_1 \wedge dA_1 + \frac{2}{3} A_1^3 \right) - k \left(A_2 \wedge dA_2 + \frac{2}{3} A_2^3 \right) + |D\phi^i|^2 + \psi^i \gamma D\psi^i + \frac{1}{k^2} |\phi^i|^6 + \frac{1}{k} (\phi^i \psi^i)^2 \right]$$

Note: No continuous couplings in accordance with gravity dual
(marginal coupling fixed by the CS level k)

- ABJ generalization

M-theory on $AdS_4 \times S^7/Z_k$ with (discrete) torion C_3 = $U(N+l)_k \times U(N)_{-k}$ Chern-Simons-Matter

for $k \ll N^{1/5}$ and large N and $G_4 = dC_3 = 0$

- $H_3(S^7/Z_k, Z) = Z_k$; $\int_{S^3/Z_k \subset S^7/Z_k} C_3 = \frac{l}{k}$ (with $l = 1, \dots, k-1$)

representing l fractional M2-branes = M5-branes wrapped on S^3/Z_k

Type IIA on $AdS_4 \times CP^3$ with NSNS B_2 = $U(N+l)_k \times U(N)_{-k}$ Chern-Simons-Matter

for $N^{1/5} \ll k \ll N$ and large N and $H_3 = dB_2 = 0$

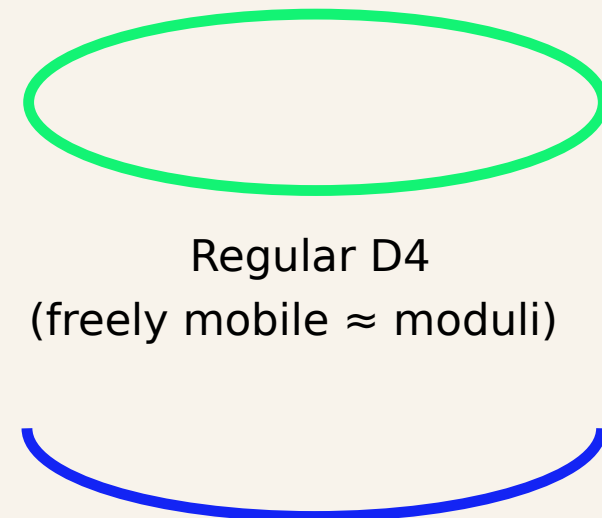
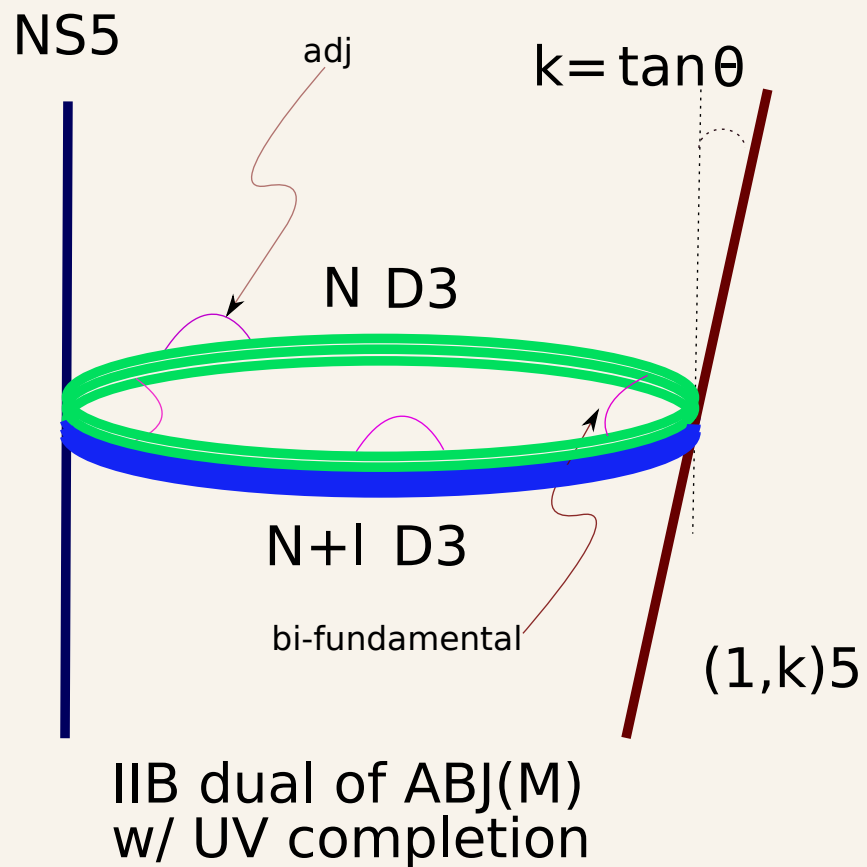
- $\int_{CP^1 \subset CP^3} B_2 = \frac{l}{k}$ (with $l = 1, \dots, k-1$)

representing l fractional D2-branes = D4-branes wrapped on CP^1

- UV completion \neq 3d $\mathcal{N} = 8$ SYM

The standard UV completion of M-theory on $AdS_4 \times S^7$
 = near-horizon D2-branes on R^7 (dual to 3d $\mathcal{N} = 8$ SYM)

- There are many different UV completions (universality) & there is one particularly useful completion:



(1) T-dual + M-theory lift \implies M2-branes on Lee-Weinberg-Yee (LWY) space (\sim nontrivially fibered double TN & toric hyper-Kähler)

$$ds_{LWY}^2 = V_{ij} d\vec{x}^i d\vec{x}^j + (V^{-1})^{ij} (d\varphi_i + A_i)(d\varphi_j + A_j)$$

with $V_{ij} = \delta_{ij} + \frac{k}{2|\vec{x}^1 + k\vec{x}^2|}$ and $A_{1,2}$ vector potential for monopole

(2) Near the core (corresponding to IR of field theory), LWY space becomes C^4/Z_k

\implies (near-horizon of) M2-branes on near-core LWY = $AdS_4 \times S^7/Z_k$

(3) Low energy effective theory on D3-branes (UV completion):

$$\mathcal{N} = 3 U(N) \times U(N+1) \text{ SYM}_{2+1} + \text{level } (k, -k) \text{ CS}$$

with (in 4d language) $\mathcal{N} = 2$ (plus $\mathcal{N} = 1$) vector multiplets + $\mathcal{N} = 1$ bi-fundamental chiral multiplets

(4) Vector multiplets massive due to CS terms ($m \sim g_{YM}^2 k$)

\implies Vector multiplets integrated out in IR, leaving $\mathcal{N} = 6$ CSM theory

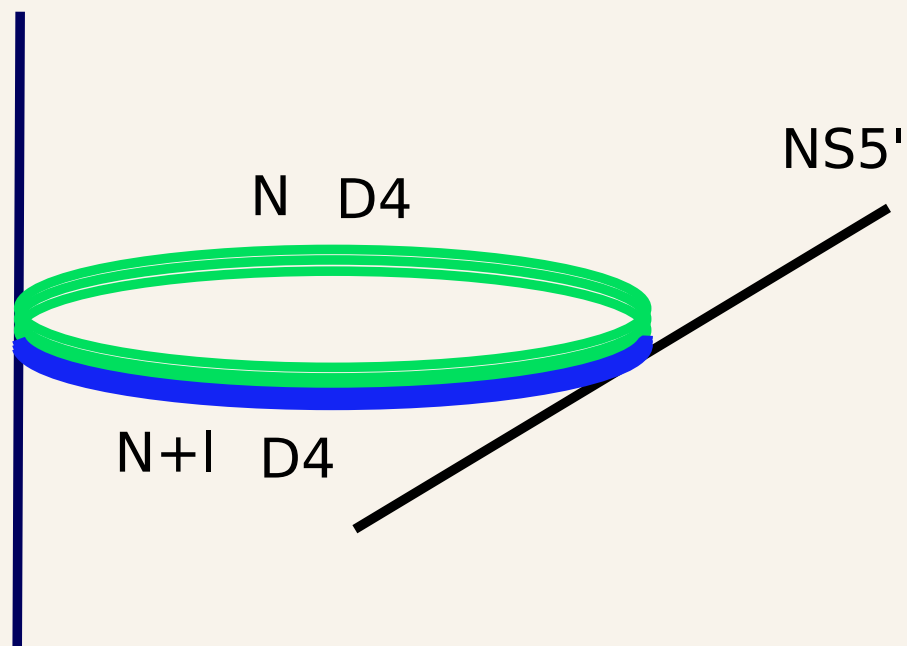
M-theory on near-horizon M2-branes
on LWY space with G-flux $= U(N+l)_k \times U(N)_{-k}$ SYM+CS

Duality Cascade

Note: similarity between UV completed ABJ and Klebanov-Strassler (KS)

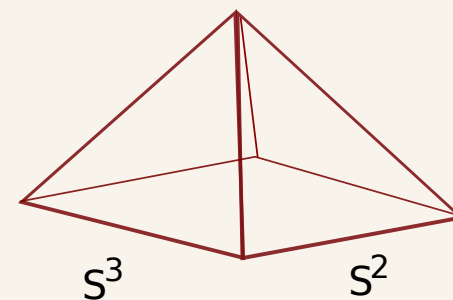
- KS theory (4d)

NS5

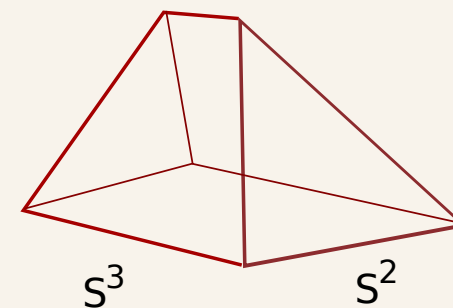


T-dual of D3 on conifold

conifold



deformed conifold



Note: Confining – dynamically generated mass gap = size of S^3 of deformed conifold (generation of mass gap \longleftrightarrow singularity resolution)

- 4d $\mathcal{N} = 1$ $SU(N)_1 \times SU(N + l)_2$ SYM with bi-fundamental matters $A_{1,2}, B_{1,2}$ with quartic potential $\text{Tr}(A_i B_j A_k B_l) \epsilon^{ik} \epsilon^{jl}$:

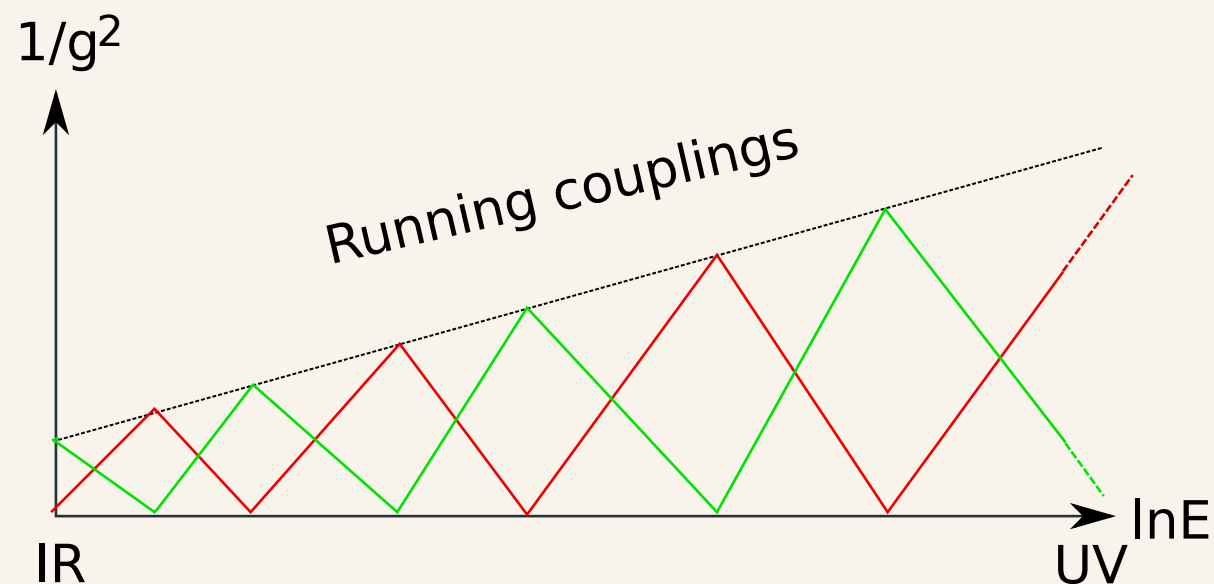
$$\beta_1^{1-loop} \sim 2lg_1^3 > 0, \quad \beta_2^{1-loop} \sim -3lg_2^3 < 0$$

(0th) $SU(N)_1$ theory IR free, while $SU(N + l)_2$ theory asymptotic free

\implies Seiberg dual $SU(N)_1 \times SU(N - l)_2$

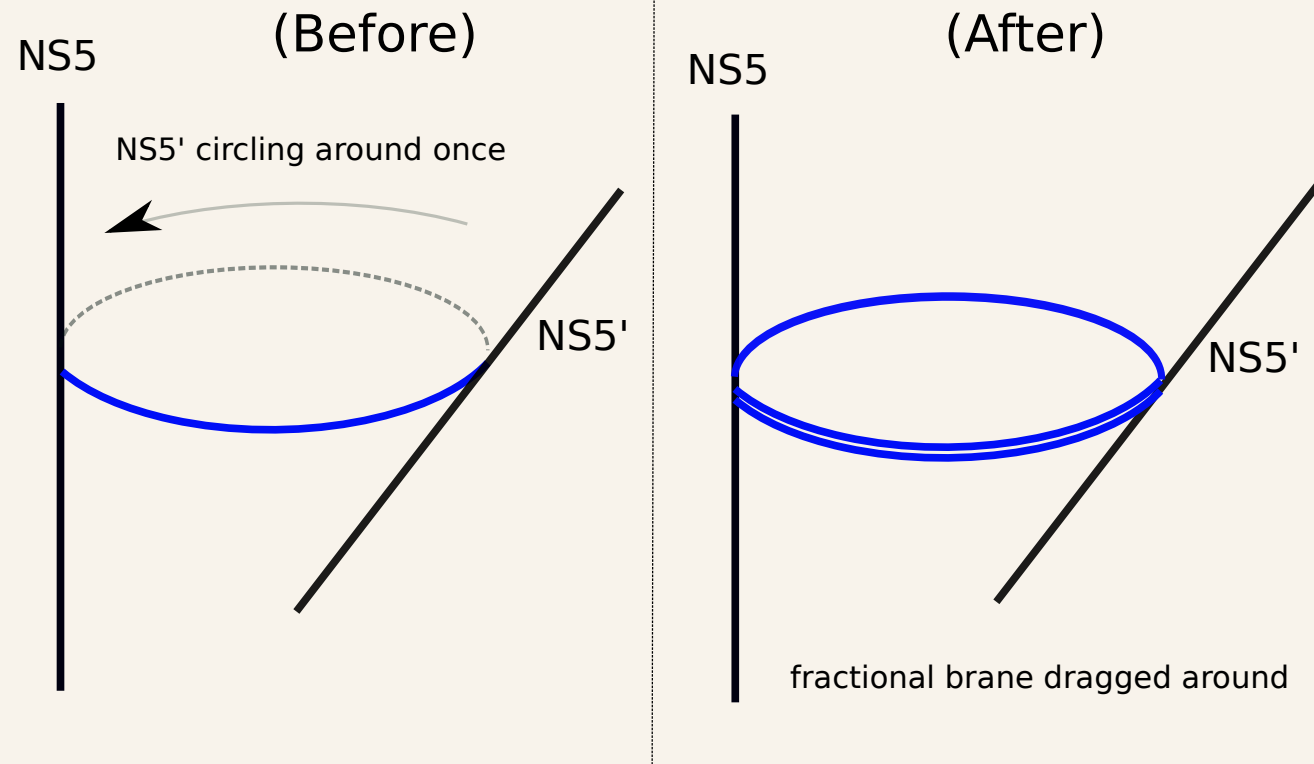
(1st) $SU(N)_1$ theory asymptotic free, while $SU(N - l)_2$ theory IR free

\implies Seiberg dual $SU(N - 2l)_1 \times SU(N - l)_2$

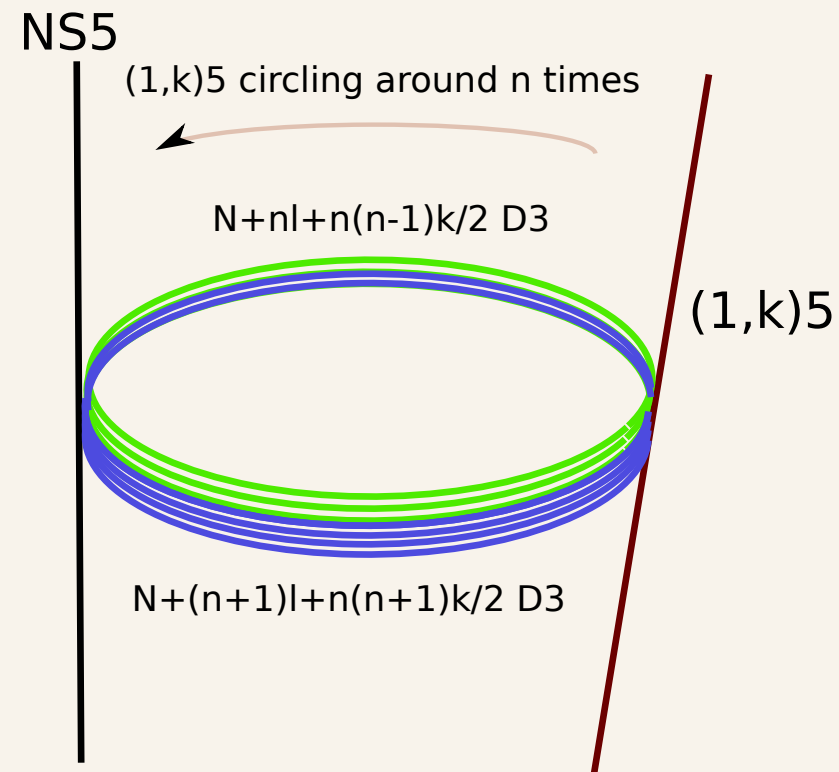


This is the duality cascade = an alternating sequence of Seiberg dualities

- duality cascade = NS5' moving around on S^1



- Duality cascade in 3d SYM + CS



- # of D3-branes changes by
 - (i) dragging D3-branes around
 - (ii) Hanany-Witten effect (when D5-branes cross NS5, D4 branes created)

Gravitational Dual (of $\mathcal{N} = 3 U(N + l) \times U(N)$ SYM + CS)

- Ansatz (M2 + wrapped M5 in 8d LWY space)

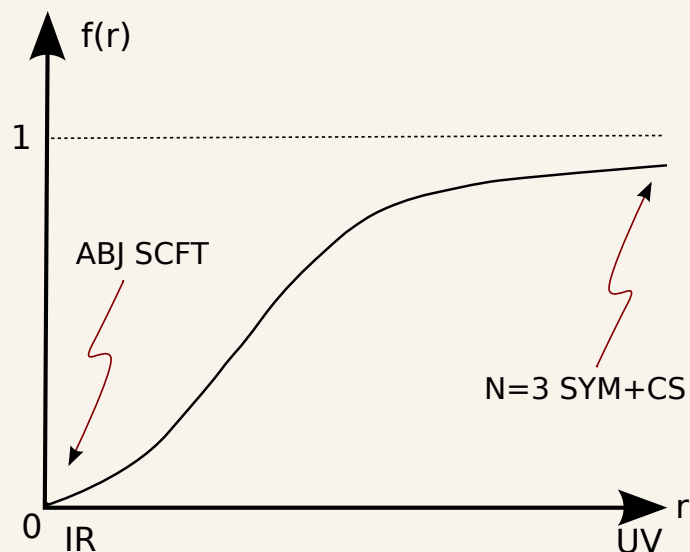
$$ds_{11}^2 = H(r)^{-2/3} ds_{R^{1,2}}^2 + H(r)^{1/3} ds_{LWY}^2$$

$$G_4 = dt \wedge dx_1 \wedge dx_2 \wedge dH(r)^{-1} + G$$

Note : G -flux (representing wrapped M5) needs to be turned on ($G = dC_3$):

$$C_3 = (k b_\infty + \hat{N}_4) f(r) (\text{nontrivial 3-form}) - \hat{N}_4 J \wedge dx_{11}$$

with $J =$ Kähler form on CP^3 (LWY \sim cone over S^1 bundle over CP^3)



- SUSY \implies G (anti-)self-dual on 8d LWY space for (anti-)M2-branes

- Two notions of charges in Type IIA SUGRA (Maxwell & Page)

Charge conservation: (spacial integration over a Gaussian surface)

$$d * F_{p+2} = *j_{p+1} \quad \Longrightarrow \quad Q_p = \int_{S_{8-p}} *F_{p+2}$$

Type IIA EOM (show only for F_4)

$$d\tilde{F}_4 = -F_2 \wedge H_3 + *j_5^{brane} \quad (\text{for D4})$$

$$d * \tilde{F}_4 = \tilde{F}_4 \wedge H_3 + *j_3^{brane} \quad (\text{for D2})$$

where

$$\tilde{F}_4 = F_4 - A_1 \wedge H_3$$

$$*j_5^{brane} = *j_5^{D4} + B_2 \wedge *j_7^{D6}, \quad *j_3^{brane} = *j_3^{D2} + B_2 \wedge *j_5^{D4} + \frac{1}{2} B_2 \wedge B_2 \wedge *j_7^{D6}$$

These EOMs can be rewritten as

$$d\hat{F}_4 = *j_5^{D4} \quad \text{where} \quad \hat{F}_4 = \tilde{F}_4 + B_2 \wedge F_2$$

$$d\hat{F}_6 = *j_3^{D2} \quad \text{where} \quad \hat{F}_6 = * \tilde{F}_4 + B_2 \wedge \hat{F}_4 - \frac{1}{2} B_2 \wedge B_2 \wedge F_2$$

(1) Maxwell charge (**NOT quantized & varies with r** due to bulk contribution)

$$Q_2^{Maxwell} = \int_{S_6} * \tilde{F}_4 \quad (\text{for D2})$$

$$Q_4^{Maxwell} = \int_{S_4} \tilde{F}_4 \quad (\text{for D4})$$

Note: D2 and D4 are electromagnetic dual to each other

(2) Page charge (**quantized**; counts # of D-branes)

$$Q_2^{Page} = \int_{S_6} \hat{F}_6 \quad (\text{for D2})$$

$$Q_4^{Page} = \int_{S_4} \hat{F}_4 \quad (\text{for D4})$$

Note: Page charges are not invariant under gauge transformation $B_2 \rightarrow B_2 + d\Lambda_1$, whereas Maxwell charges are. **The gauge dependence of Page charges is closely related to Seiberg-like duality and cascade**

- $kb_\infty + \hat{N}_4$ and \hat{N}_4 fixed by dimensional reduction to type IIA:

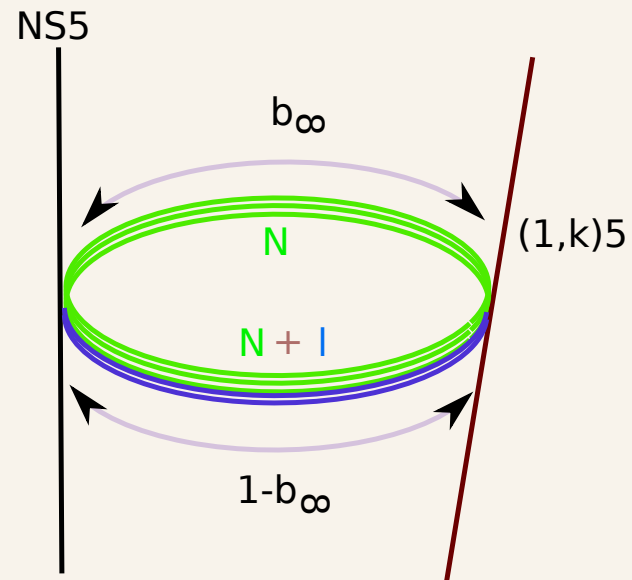
$$\text{(Note: } C_3 = A_3^{RR} - B_2 \wedge dx_{11}\text{)}$$

- (1) exact (singular) part \implies fractional D2 (wrapped D4) quantization

$$\hat{N}_4 = Q_4^{Page} = \int_{CP^2} (\tilde{F}_4 + (B_2 + F) \wedge F_2) = l - \frac{k}{2}$$

- (2) nontrivial part \implies separation between NS5 and (1,k)5

$$b_\infty = \int_{CP^1} B_2(\infty)$$



Note : D2 and D4 **Maxwell charges** precisely account for the cascade pattern:

$$b(r) \rightarrow b(r)+n \quad \Longrightarrow \quad \begin{cases} Q_2^{Maxwell} = \int_{CP^3} * \tilde{F}_4 \rightarrow N + nl + \frac{1}{2}n(n-1)k \\ Q_4^{Maxwell} = \int_{CP^2} \tilde{F}_4 \rightarrow l + nk \end{cases}$$

- To be fully explicit, we need to find the **self-dual 4-form** G on the LWY space. This problem remains to be solved (though Gibbons claimed otherwise). Once we know it, the warp factor can be found

$$d *_8 dH = -\frac{1}{48}|G|^4 dV_{LWY}$$

SUSY breaking

- Witten's conjecture

SUSY dynamically broken if $|k| < N/2$ in $\mathcal{N} = 1$ $SU(N)$ SYM + level k CS

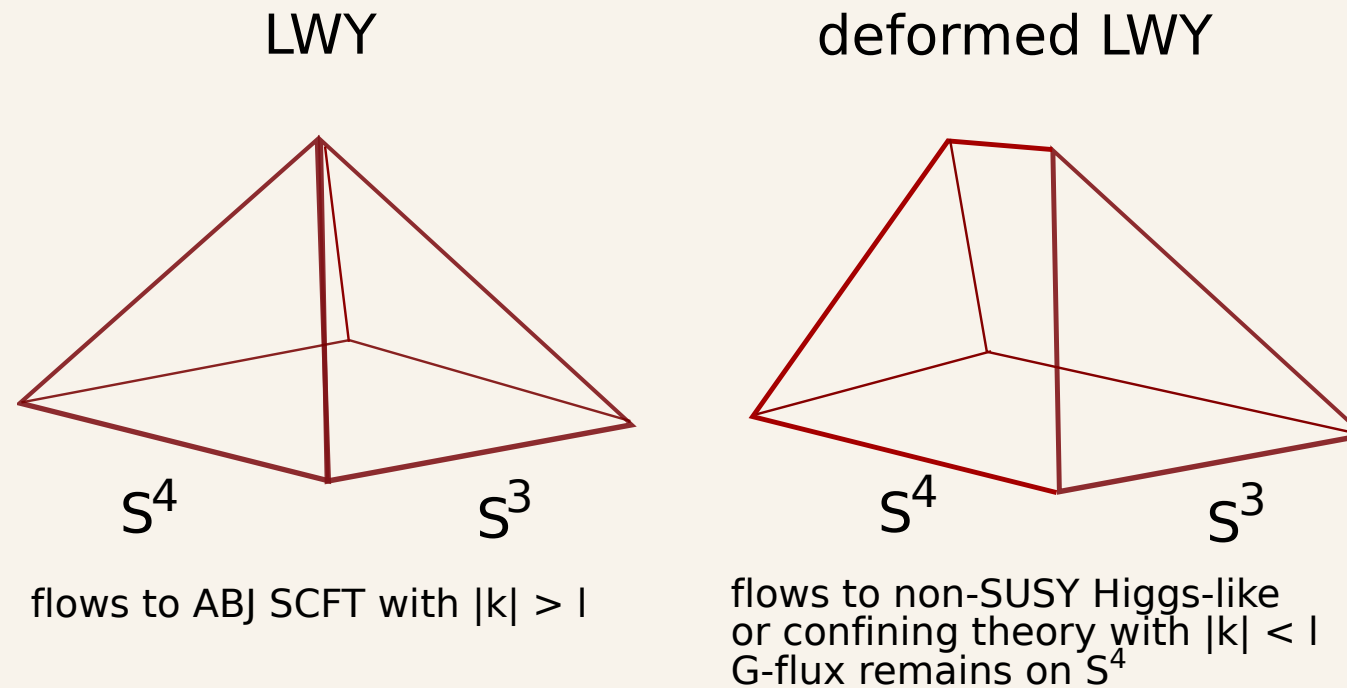
where Witten index $I(k) = \frac{1}{(N-1)!} \prod_{j=-N/2+1}^{N/2-1} (k - j)$

- When applied to ABJ-type theories,

SUSY dynamically broken if $|k| < l$

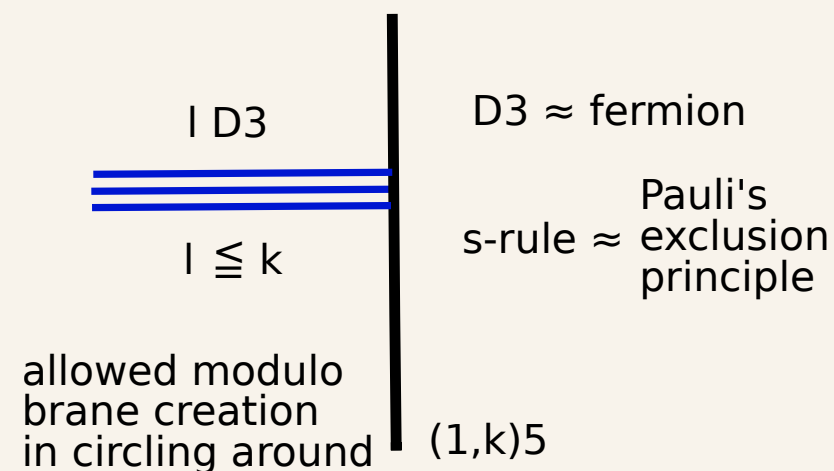
- This implies $\mathcal{N} = 3$ SYM + CS theory with $|k| < l$ cannot flow to superconformal ABJ theory. As a corollary ABJ's $U(N+l)_k \times U(N)_{-k}$ CSM theory with $l > |k|$ does not exist as a unitary theory

- What may happen is expected to be the following:



where the **SUSY breaking scale = size of S^4** (analogous to the mass gap in KS)

- The SUSY breaking condition $|k| < l$ is closely related to the s-rule
(generalized) s-rule



- A closer look – One can show that

$$Q_2^{Maxwell}(\text{IR}) < 0 \iff Q_4^{Maxwell}(\text{IR}) > k$$

The SUSY breaking condition $|k| < l$ is equivalent to having **negative** D2-brane charge. In other words, the SUSY is broken because of the presence of **anti** D2-branes (in the background of D2-branes).

Dynamical SUSY breaking of ABJ-type theories = adding anti D2-branes

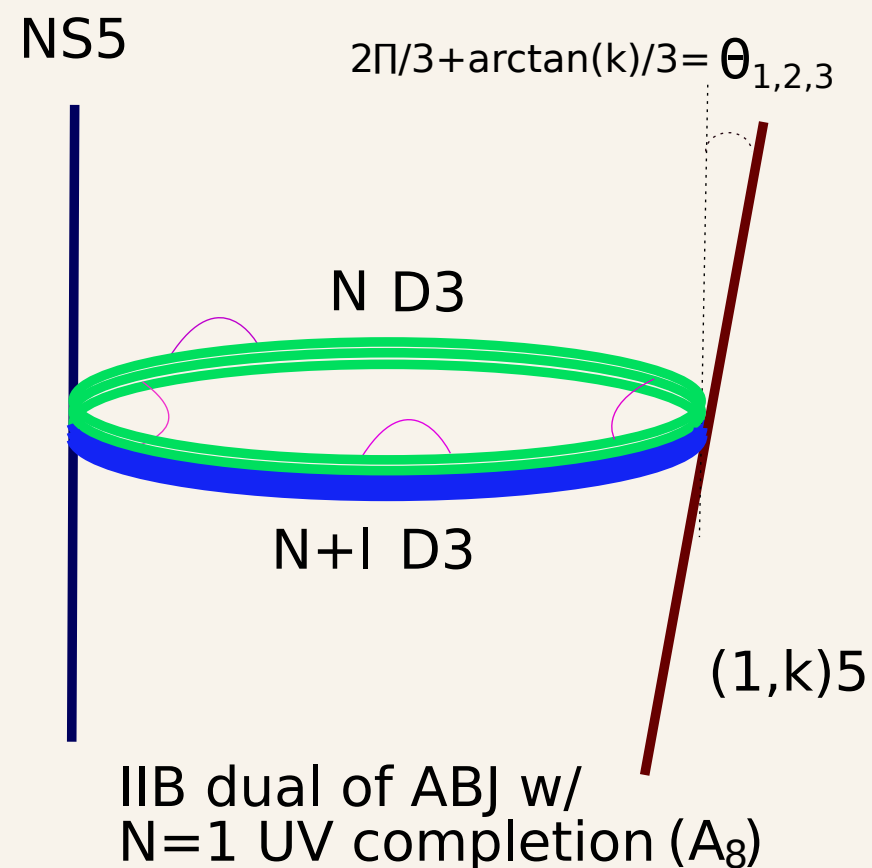
Anti-climatic but turns out to be as trivial as this

Search for Gravity Dual of SUSY Breaking Phase

- In order to construct gravity dual of DSB, we first need to firmly understand gravity dual of SUSY phase. The difficulty of constructing the self-dual 4-form on LWY space is hard to overcome

- UV completion w/ $\mathcal{N} = 1$ SUSY:

Consider yet another UV completion of ABJ ($Spin(7)$ manifold A_8)



- The only **difference** between $\mathcal{N} = 3$ and this $\mathcal{N} = 1$ cases is the **rotation angles** $\theta = \theta_{1,2,3}$. We expect to gain qualitative understanding of DSB in $\mathcal{N} = 3$ SYM + CS from $\mathcal{N} = 1$ theory

- Advantage: (anti-)self-dual 4-form explicitly constructed
- Disadvantage: Dual $\mathcal{N} = 1$ field theory poorly understood

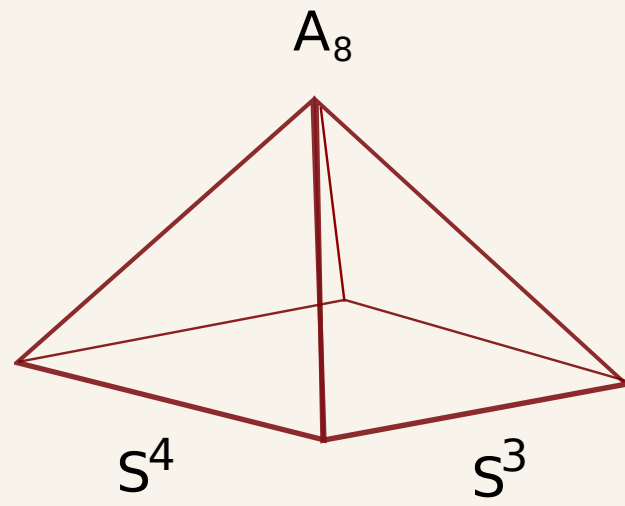
- *Spin*(7) manifolds

ALC (asymptotically locally conical) cones over squashed S^7 (Cvetic-Gibbons-Lu-Pope) \sim analogous to Taub-NUT and Taub-BOLT

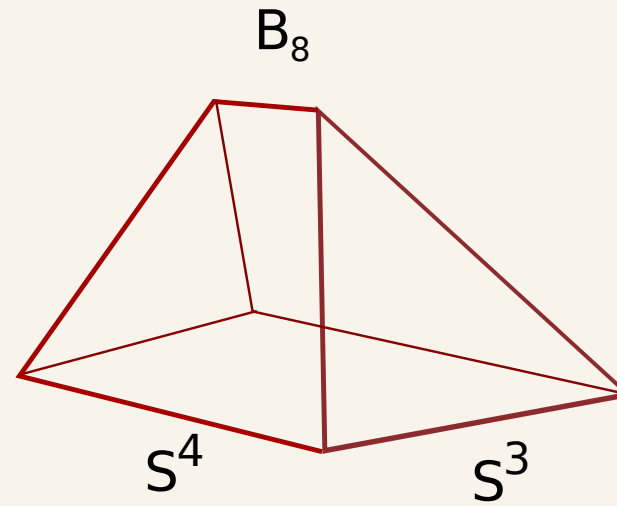
$$ds^2 = \frac{(r \pm \ell)^2}{(r \pm 3\ell)(r \mp \ell)} dr^2 + \frac{1}{4}(r \pm 3\ell)(r \mp \ell) (D\mu_i)^2 + \frac{\ell^2(r \pm 3\ell)(r \mp \ell)}{(r \pm \ell)^2} \sigma^2 + \frac{1}{2}(r^2 - \ell^2) d\Omega_4^2$$

(+) A_8 manifold : R^8/Z_k (in IR _{$r=\ell$} of gauge theory)
(cone over squashed CP^3) \times S^1 (in UV _{$r=\infty$})

(-) B_8 manifold : $R^4/Z_k \times S^4$ (in IR _{$r=3\ell$})
(cone over squashed CP^3) \times S^1 (in UV _{$r=\infty$})



flows to ABJ SCFT with $|k| > 1$



flows to ??
G-flux remains on S^4

- A closer look:

$$\Delta H = -\frac{|G|^2}{48} - \left[Q_2^{Maxwell}(\text{IR}) + \frac{C^2}{2k} \left(kb_\infty + N_4 - \frac{k}{2} \right)^2 \right] \delta^8(x)$$

where G self-dual and $Q_2^{Maxwell}(\text{IR}) = N_2 - \frac{N_4(N_4-k)}{2k}$, and $C = 0$ for A_8 and $C \neq 0$ for B_8

- For A_8 , free of repulson singularity ($H > 0$ for all $r \geq \ell$) only if

$$Q_2^{Maxwell}(\text{IR}) \geq 0$$

Note! This coincides with the SUSY bound (no anti M2-branes)

- IR geometry ($r \rightarrow \ell$)

(i) $Q_2^{Maxwell}(\text{IR}) > 0$ ($|k| > l$) $\implies AdS_4 \times S^7/Z_k$ (ABJ SCFT)

(ii) $Q_2^{Maxwell}(\text{IR}) = 0$ ($|k| = l$) $\implies R^{1,2} \times R^8/Z_k$ (SUSY w/ mass gap)

(iii) $Q_2^{Maxwell}(\text{IR}) < 0$ ($|k| < l$) \implies repulson singularity

This suggests

generation of SUSY breaking scale/mass gap \iff singularity resolution

Analogy: In KS, confinement resolves the singularity

- Curiously, SD 4-form is SUSY-breaking on B_8 + same leading UV asymptotics with that of A_8 (both geometry and flux)

$$??(iii)?? \quad Q_2^{Maxwell}(\text{IR}) < 0 \implies \text{IR } R^{1,2} \times R^4/Z_k \times S^4$$

(SUSY broken w/ mass gap)

- **Caveat:** Too opportunistic. Can't rule out the possibility that warped B_8 with SD 4-form differs from A_8 with SD 4-form by SUSY breaking relevant deformations.

- It is perhaps more likely that we need to consider SUGRA solutions with more relaxed ansatz:

$$ds^2 = e^{2A} ds_{R^{1,2}}^2 + e^{2B} ds_{A_8 + \delta A_8}^2$$

$$G_4 = dt \wedge dx_1 \wedge dx_2 \wedge de^F + G \quad \text{w/ (non-)SD } G$$

In other words the gravity dual of DSB may not be as simple as warped B_8 geometry with (SUSY breaking) SD 4-form (Perhaps, SUSY breaking B_8 is more like skew-whiffing?)

Summary

- The refined understanding of $\text{AdS}_4/\text{CFT}_3$ is one of the recent major breakthroughs in string theory (BLG + ABJ/M theories)
- Discussed gravitational descriptions of (2+1)d supersymmetric Yang-Mills + Chern-Simons theories which flow in the IR to ABJ(M)'s superconformal Chern-Simons-matter theories and undergo the duality cascade
- Discussed an attempt to construct gravity duals of dynamical SUSY breaking expected to occur in the SYM + CS theories

Nagoya University Global COE Program

International Spring School 2011

Gauge Theory, Gravity, and String Theory

Date: March 21-24, 2011

Place: School of Science, Nagoya University

Web: [site:http://www.gcoe.phys.nagoya-u.ac.jp/ggs2011/](http://www.gcoe.phys.nagoya-u.ac.jp/ggs2011/)

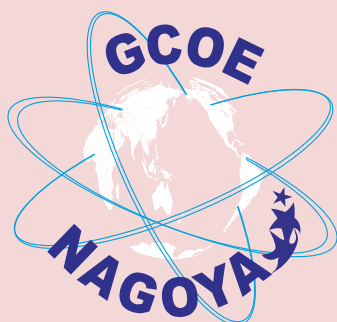
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Sumit Das (Univ. of Kentucky)

Nadav Drukker (Imperial College)

Romuald Janik (Jagiellonian Univ.)



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