Rolling Tachyon and S-brane Actions

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Open string tachyon condensation is established via Sen’s conjectures to be a very important tool for describing D-branes. It opens up possibilities to describe dynamical creation and annihilation processes of D-branes.

Rolling tachyons

It is known that spacelike-branes (S-branes) appear in the rolling tachyon.

After reviewing some, I would like to present

- Derivation of S-brane actions
- Application of S-brane actions:
  - Formation of a closed string in rolling tachyon
  - Formation of a lower-dimensionaional D-brane
  - D-brane as a boosted S-brane
Sen studied how a non-BPS D-brane decays dynamically, with time-dependent tachyon condensation. His approach to the dynamical decay of unstable D-branes:

- Treating classical decay without coupling to closed strings.
- Using SFTs, deformed CFTs and boundary states.

Results:

- No oscillation around the potential minimum, but everlasting rolling toward it.
- End product is pressure-less non-interacting dust with finite energy density, which is called “tachyon matter.”
- Proposal of an effective field theory for the tachyon matter.
Tachyon Action

Start with an effective Lagrangian of a non-BPS D(p+1)brane:

\[ S = - \int d^{p+2}x \ V(T) \sqrt{1 + (\partial_\mu T)^2}, \]

where the potential is the run-away type for large \( T \),

\[ V(T) = \exp[-\alpha T], \quad \alpha = 1, \sqrt{2}. \]

- Proposed by \[ \text{Garousi}(0003122) \] and \[ \text{Bergshoeff-deRoo-deWit-Eyras-Panda}(0003221) \], so that it is consistent with scattering amplitudes and T-duality.
- \[ \text{Sen}(0203265,0204243) \] used this Lagrangian to show the appearance of the rolling tachyon classically,

\[ \dot{T} = 1 \ (x^0 \rightarrow \infty) \]

and reproduction of the exponential fall off of the pressure.
- BSFT reproduces Sen’s results mostly. \[ \text{Sugimoto-Terashima}(0205085), \text{Minahan}(0205098) \]
Sp-brane Solution

\[
\text{Sp-brane} = \text{Time-dep. kink in non-BPS D}(p+1)\text{-brane}
\]

Assume a homogeneous tachyon: \( T = T(x^0) \)

\[
\downarrow
\]

Conserved energy: \( \mathcal{E} = \frac{V(T)}{\sqrt{1-T^2}} (> V(0)) \)

\[
\downarrow
\]

A homogeneous solution \( T_{cl}(x^0) : \)

\[
x^0 = \int_0^{T_{cl}} \frac{dT}{\sqrt{1 - V(T)^2/\mathcal{E}^2}}
\]

- \( V(T) \to 0 \ (T \to \infty) \)
  \( \Rightarrow \ T \sim x^0 \) (rolling tachyon)

- An \( Sp \)-brane sits at \( x^0 = 0 \) where \( T_{cl}(x^0) \) passes its potential maximum \( T = 0 \).

- This S-brane has a RR-charge. \( S_{CS} \propto \int e^{-T^2}dT \wedge C \)
  \( \rightarrow \) S-brane \( \sim \) Spacelike D-brane

\[
T > 0
\]

\[
S\text{-brane}
\]

\[
T < 0
\]

\[
\text{non-BPS brane}
\]
What is the S-brane “dynamics”?
What is the role of the S-branes in string theory?

A hint: Low energy effective actions for S-branes — S-brane actions

S-brane actions (1): Lowest order in a scalar field

Zero mode associated with the breaking of the time translation:

\[ T = T_{\text{cl}}(x^0) + t(x^\mu) \]

with \( t(x^\mu) = X^0(x^{\hat{\mu}}) \dot{T}_{\text{cl}}(x^0) \)

\( (\hat{\mu} = 1, 2, \cdots, p + 1) \)

\[ \downarrow \] Substitution to the tachyon action

\[ S = -\mathcal{T}(\mathcal{E}) \int d^{p+1}x^\hat{\mu} \frac{1}{2} (\partial_{\hat{\nu}} X^0)^2 \]

S-brane effective action to the lowest order.

- Action is defined on a \( p+1 \) dimensional Euclidean space.
- \( X^0 \): Scalar field for displacement along time direction.
Step 1. The dependence on zero mode $X^0$ in tachyon action is

$$S = -\int d^p x \, L \left( T_{\text{cl}} \left( \frac{x^0 + X^0(x^\mu)}{\beta(X^0)} \right) \right).$$

Here $\beta(X^0)$ can be fixed by the global Lorentz invariance in the world volume spacetime.

[Arutyunov-Frolov-Theisen-Tseytlin](0012080)

Step 2. We turn on constant gauge field strength as a background on the non-BPS brane. It appears in the tachyon as

- $\eta^{\mu\nu}$ is replaced by open string co-metric $G^{\mu\nu} = \left( (\eta + F)^{-1}_{\text{sym}} \right)^{\mu\nu}$.
- BI factor $\sqrt{-\det(\eta + F)}$ is multiplied on the Lagrangian.
- For the tachyon EOMs to be satisfied with the homogeneous profile, $G^{00} = -1, G^{0\hat{\mu}} = 0$. (Electric fields = 0)
**Step 3.** Lorentz boosts preserving the open string metric:

\[
\Lambda_\mu^\nu G_{\nu\rho}(\Lambda^t)^\rho_{\phantom{\rho}\sigma} = G_{\mu\sigma}, \quad \Lambda_\mu^\nu = \begin{pmatrix} 1/\beta & \partial_\mu X^0/\beta \\ 0 & 1 \end{pmatrix} \\
\Rightarrow \beta = \sqrt{1 - G_{\hat{\mu}\hat{\nu}} \partial_{\hat{\mu}} X^0 \partial_{\hat{\nu}} X^0}
\]

**Step 4.** Performing integration over \( x^0 \) gives the S-brane Action:

\[
S = S_0(\mathcal{E}) \int d^{p+1}x \beta(X^0) \sqrt{\det(\delta + F)}_{\hat{\mu}\hat{\nu}} \\
= S_0(\mathcal{E}) \int d^{p+1}x \sqrt{\det(\delta_{\hat{\mu}\hat{\nu}} - \partial_{\hat{\mu}} X^0 \partial_{\hat{\nu}} X^0 + F_{\hat{\mu}\hat{\nu}})}.
\]

- Action is defined on a Euclidean worldvolume.
- Kinetic term of \( X^0 \) has a “wrong” sign due to that it represents time translation.
- Inclusion of the other transverse scalars is easy:

\[
S = S_0(\mathcal{E}) \int d^{p+1}x \sqrt{\det(\delta_{\hat{\mu}\hat{\nu}} - \partial_{\hat{\mu}} X^0 \partial_{\hat{\nu}} X^0 + \partial_{\hat{\mu}} X_i \partial_{\hat{\nu}} X^i + F_{\hat{\mu}\hat{\nu}})}.
\]

- This suggests that the general form is written with the induced metric

\[
S = S_0(\mathcal{E}) \int d^{p+1}x \sqrt{\det(g_{\hat{\mu}\hat{\nu}} + F_{\hat{\mu}\hat{\nu}})}, \quad \text{where } g_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} X^M \partial_{\hat{\nu}} X^N G_{MN}
\]

\( \rightarrow \) Consistent with CFT derivation.
S-brane Descent Relations

Array (1) : Formation of a time-dependent kink, which is an S-brane.

Array (2) : Formation of a time-dependent kink in $D\bar{D}$. This indicates a new object which is named as “tachyonic S-brane”.

Array (3) and the other vertical short arrows : Formation of an ordinary tachyon kink (space-dependent).

Array (4) : Formation of a space-time vortex in $D\bar{D}$, which should lead to the S-brane.

Vertical long arrows : Formation of a vortex whose co-dimension is two.

Left and Right sequences are related by Euclideanization.
3 Application of the S-brane Actions

We will find that S-branes are very useful to describe remnants of tachyon condensation, formation of defects in rolling tachyons.

ex.) [S-branes : $T \sim 0] \leftrightarrow$ D-branes and strings in noncommutative tachyons

ex.) Solution of the defect action $\rightarrow$ Tachyon configuration

[Hirano-K.H.](0102173,0102174)

Solution of a D3-brane action representing a Non-commutative Dirac monopole:

\[
X = c/r + \theta_{23} x^1
\]
\[
B_i = \partial_i X
\]

with $r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$

\[
\downarrow
\]

Corresponding tachyon configuration:

\[
T = uX \quad \text{with} \quad u \rightarrow \infty
\]
\[
B_i = \partial_i X
\]

Also in the S-brane case, this kind of correspondence is expected.
3-1. Formation of a F-string in rolling tachyon

(1) How the spacelike S-branes can be transformed into timelike?

S-brane action with $F = 0$:

$$\sqrt{1 - (\nabla X^0)^2}$$

$$|\nabla X^0| \leq 1 \Leftrightarrow |v| = |\nabla X^0|^{-1} \geq 1 : \text{S-brane is spacelike.}$$

However, we can make this S-brane timelike as follows.

Turn on a single gauge potential $A_{p+1}$ and suppose that all the worldvolume fields are independent of $x^{p+1}$.

S-brane action becomes

$$\sqrt{\det(\delta_{\hat{\mu} \hat{\nu}} - \partial_{\hat{\mu}} X^0 \partial_{\hat{\nu}} X^0 + \partial_{\hat{\mu}} A_{p+1} \partial_{\hat{\nu}} A_{p+1})}$$

Static DBI with replacement $(X^0, A_{p+1}) \leftrightarrow (A_0, \Phi)$.

Spike solutions exist!

[Callan-Maldacena], [Gibbons]
(2) **Spike solution**

In the $S^p$-brane (for $p \geq 3$), the solution is

$$X^0 = A_{p+1} = \frac{c_p}{r^{p-2}} \quad (r = \sqrt{(x^1)^2 + \cdots + (x^p)^2})$$

- $r$ is parameterizing also the time evolution of this S-brane.
- As time evolves the radius decreases.
  - Remnant becomes 1+1 dimensional object (string) parametrized by $X^0$ and $x^{p+1}$.

- Induced metric on the S-brane:

$$ds^2 = (dx^{p+1})^2 + r^2 d\Omega^2_{p-1} + \left(1 - \left(\frac{dX^0(r)}{dr}\right)^2\right) dr^2$$

  Euclidean for $0 < X^0 < X^0_c$,
  Worldvolume is timelike for $X^0_c < X^0$!

where $X^0_c \equiv \frac{c_p}{(p-1)} \frac{(p-2)}{(2-p)/(p-1)}$. 
(3) The remnant = F-string

[a.] (Induced critical electric field)

\[ \dot{T} = 0 \]

\[ \dot{T} \sim 1 \]

\[ x^{p+1} \]

\[ \frac{\partial}{\partial X^0} F_{rp+1} = \frac{1}{2\pi\alpha'} \]

The field strength induced on the deformed S-brane:

\[ F_{0p+1} = \frac{\partial r}{\partial X^0} F_{rp+1} \]

Critical electric field indicating induced F-string charge

This criticality is consistent with rolling tachyon with flux, \( \dot{T}^2 + E^2 = 1 \) with \( \dot{T} = 0 \) at late time.

[Sen-Mukdpadhyay] [Gibbons-K.H.-Yi]

Confined electric flux tube
[b.] ⟨F-string tension⟩

In coordinates more appropriate to spacetime viewpoint,

\[ S = S_0 \int dx^0 d^p x \sqrt{-1 + E_{p+1}^2 + \dot{r}^2}, \]

Canonically conjugate momenta and Hamiltonian:

\[ D = S_0 \frac{E}{-1 + E^2 + \dot{r}^2}, \quad P_r = S_0 \frac{\dot{r}}{-1 + E^2 + \dot{r}^2}, \quad H = \frac{S_0}{\sqrt{-1 + E^2 + \dot{r}^2}} = \frac{D}{E}. \]

\[ \left[ \text{Flux quantization} \right] \int d^{p-1} x \ D = n \quad + \quad \left[ \text{Critical} \ E = 1 \right. \]

at late times

\[ \downarrow \]

\[ \int d^p x \ H = \frac{n}{2\pi\alpha'} \int dx^{p+1} \]

F-string tension
It is natural to take the Ramond-Ramond (RR) coupling of the S-brane to be the same as that for a D-brane. Transforming $r$ into the embedding time $X^0$ we obtain

$$\mu \int C = \mu \int C^{p+1} r \Omega r^{p-1} dx^{p+1} dr d\Omega_{p-1}$$

$$= \mu \int C^{p+1} 0 \Omega \left( \frac{X^0}{c_p} \right)^{-\frac{p-1}{p-2}} dX^0 d\chi d\Omega_{p-1}$$

Due to the factor $(X^0)^{-(p-1)/(p-2)}$ which goes to zero at late times, we see that this has the D-brane charge which shrinks to zero at the future infinity.

This is similar to the F-string popped-up into a cylindrical D-brane with electric fields, studied by Emparan.

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[a][b][c] ⇒ The remnant = F-string
3-2. Formation of a lower dimensional D-brane

(1) [S-duality of S3-brane] + [Spike solution above]

↓

Spike solution representing a formation of a D-string from a non-BPS D4-brane (an S3-brane)
(S-duality for an S3-brane was already shown by Tseytlin.)

(2) Formation of a D0-brane from an S2-brane (non-BPS D3-brane)

Lagrangian for a S2-brane with $B_i \equiv \epsilon_{ijk}F_{jk}/2$:

$$L = \sqrt{1 - |\nabla X^0|^2 + |B|^2 - (B \cdot \nabla X^0)^2} - i\nabla \chi \cdot B.$$  

$\rightarrow$  

$$L = \sqrt{\det(\delta_{ij} - \partial_i X^0 \partial_j X^0 + \partial_i \chi \partial_j \chi)}.$$  

$\sim$ SM2-brane

Self-dual configuration of duality-R symmetry $SO(1, 7)$:

$$X^0 = \chi = \frac{c}{r}, \quad r \equiv \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$$

Formation of a D0-brane (= a shrinking dielectric brane)
3-3. D-brane as a boosted S-branes

For $p = 1$ we obtain a moving $(p,q)$-string.

$$\partial_1 X^0 = \frac{c_1}{\sqrt{1 - c_2^2 + c_1^2}}, \quad F_{12} = \frac{c_2}{\sqrt{1 - c_2^2 + c_1^2}}.$$ 

- The speed along $x^1$: 
  $$|v_1| = |(\partial_1 X^0)^{-1}| > 1/\sqrt{1 + F_{12}^2},$$ 
  which has a timelike region.

- Why this is possible? 
  The open string light cone lies always inside the closed string light cone. [Gibbons-Herdeiro](0008052)

- A boundary state describing this → it becomes a source for the NS-NS B-field, suggesting that the moving object is a $(p, q)$-string.

- Corresponding tachyon solution with $F_{12} \to \infty : T = u x^1$ with $u = \infty$. 

[Diagram showing light cones and moving D-brane]

open string lightcone

moving D-brane

closed string lightcone
Corresponding tachyon solution

A non-BPS D2-brane:

\[ L = -V(T) \sqrt{-\det(\eta + F)} \mathcal{F}(G^{\mu\nu} \partial_\mu T \partial_\nu T) \]

The open string metric with \( F_{12} \) is \( G_{\mu\nu} = \text{diag}(-1, 1 + (F_{12})^2, 1 + (F_{12})^2) \)

The simplest homogeneous solution: \( T = T_{\text{cl}}(x^0) \).

Let us make a boost which preserves the metric. A rescaled coordinate \( \tilde{x}^1 \equiv \sqrt{G_{11}} x^1 \),

\[
\begin{pmatrix} x^0 \\ \tilde{x}^1 \end{pmatrix} \rightarrow \begin{pmatrix} x^0' \\ \tilde{x}'^1 \end{pmatrix} = \begin{pmatrix} \cosh \theta & \sinh \theta \\ \sinh \theta & \cosh \theta \end{pmatrix} \begin{pmatrix} x^0 \\ \tilde{x}^1 \end{pmatrix}.
\]

Location of the S1-brane: \( x^0 = 0 \) \( \rightarrow \) \( x^0 + \tanh \theta \sqrt{G_{11}} x^1 = 0 \)

The defect is now moving along \( x^1 \) with the velocity:

\[ \frac{\partial x^1}{\partial x^0} = \frac{-1}{\sqrt{G_{11}} \tanh \theta}. \]

Take the limit \( F_{12} \rightarrow \infty \).

\[ T = T_{\text{cl}} \left( x^0 + \tanh \theta \sqrt{G_{11}} x^1 \right) \rightarrow T \sim \tanh \theta F_{12} x^1 \]

D-brane solution in BSFT
Conclusion and Future Directions

Summary

- S-brane actions are constructed.
- Dynamical formation of tachyon remnants is described.
  - Spike solutions exist.
  - They describe formation of a F-string/D-string in a non-BPS D4-brane.
  - D0-brane formation = Shrinking dielectric brane.
  - Moving D-brane = Boosted S-brane.

Future directions

- Much to be studied with use of the S-brane actions!
  - Various brane configurations such as S-brane junctions and spherical S-branes
  - Their supersymmetric properties — Their M-theory lift — S F -strings
  - Matrix theory with S-instantons — S/T-duality on S-brane actions
  - Noncommutative S-branes — Non-Abelian S-brane actions etc....

- $g_s \neq 0$?